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# Approximate Solution for Fractional Black-Scholes European Option Pricing Equation

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**Keywords:**
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**Abstract:** The Black-Scholes equation is one of the most significant mathematical models for a financial market. In this paper, the homotopy perturbation method is combined with Mohand transform to obtain the approximate solution of the fractional Black-Scholes European option pricing equation. The fractional derivative is considered in the Caputo sense. The process of the methods which produce solutions in terms of convergent series is explained. Some examples are given to show a powerful and efficient method to find approximate analytical solutions for fractional Black-Scholes European option pricing equation. Further, the same equation is solved by the homotopy perturbation Sumudu transform method. The results obtained by the two methods are in agreement.

## الحل التقريبي لمعادلة بلاك شولز من الرتبة الكسرية معادلة تسعير الخيار الاوروبي

**الكلمات المفتاحية :**
معادلة بلاك–شولز.
طريقة التشويش المضطرب.
تحويل مهند التكاملي.
تفاضل كابوتو الكسري.

**المستخلص :** تعد معادلة بلاك–شولز واحدة من أكثر النماذج الرياضية أهمية بالنسبة للسوق المالي. في هذا البحث، تم دمج طريقة التشويش المضطرب مع تحويل مهند للحصول على الحل التقريبي لمعادلة بلاك شولز للتسعير الأوروبي من الرتبة الكسرية. التفاضل الكسري يكون تحت تعريف كابوتو. سوف يتم شرح الطريقة و كيفية الحصول على الحل كمتسلسلة من الحدود المتقاربة. تم إعطاء بعض الأمثلة لإظهار قوة وفعالية هذه الطريقة لإيجاد حلول تحليلية و تقريبية لمعادلة بلاك– شولز للتسعير الأوروبي من الرتبة الكسرية. علاوة على ذلك، تم حل نفس المعادلة من خلال طريقة تحويل اضطراب هوموتوبي سومودو. النتائج التي تم الحصول عليها من خلال الطريقتين متوافقة.

## INTRODUCTION

In recent years fractional partial differential equations have received considerable interest and have been applied to many problems which are modeled in various areas for instance: several physical phenomena and economies are represented by such equations (Oldham & Spanier, 1974; Zhu et al., 2014) On the other hand, many authors studied the existence of the solution of the Black-Scholes equation (Ankudinova & Ehrhardt, 2008; Bohner & Zheng, 2009; Cen & Le, 2011; Company et al., 2008; Gülkaç, 2010).

The homotopy perturbation method was first introduced and applied by He (He, 1999, 2000, 2006). This method has been coupled with integral transforms and applied by many authors, for example, homotopy perturbation is combined with Laplace transform, Sumudu transform, and Mohand transform to solve many problems such as one-dimensional non-homogenous

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partial differential equations with a variable coefficient (Madani et al., 2011), Black-Scholes of fractional order (Elbeleze et al., 2013; Kumar et al., 2012), Klein-Gordn (Dubey et al., 2022).

The Mohand transform was first proposed and introduced by (Mohand & Mahgoub) in 2017 and applied by many authors, (Aggarwal et al., 2020; Attaweel & Almassry, 2020; Qureshi et al., 2020).

(Khan & Ansari, 2016) presented an analytical solution of the Fractional Black-Scholes European option pricing equation in the form of the Fractional Taylor series with easily computable components. On the same side (Ravi Kanth & Aruna, 2016) suggested two methods for the solution of the time fractional Black-Scholes European option pricing equation. These methods are the fractional differential transform method (FDTM) and the modified fractional differential transform method (MFDTM).

In the present paper, fractional Black-Scholes European pricing equations are obtained from the corresponding integer order equation by replacing the first-order time derivatives with a fractional derivative in the Caputo sense of order $\alpha$ with $0 < \alpha \leq 1$. This equation is described by the following equation

$$\frac{\partial^\alpha u}{\partial t^\alpha} + \frac{\sigma x^2}{2}\frac{\partial^2 u}{\partial x^2} + r(t)x\frac{\partial u}{\partial x} - r(t)u] = 0, (x,t) \in \mathbb{R}^+ \times (0,T), 0 < \alpha \leq 1 \qquad (1)$$

Where $u(x,t)$ is the European call option, price at asset price $x$ and at time $t, T$ is the maturity, $r(t)$ the risk-free interest rate, and $\sigma(x)$ represents the volatility function of the underlying asset. The payoff functions are

$$u_c(x,t) = \max(x - E, 0); \qquad (2)$$

$$u_p(x,t) = \max(E - x, 0)$$

Where $u_c(x,t)$ and $u_p(x,t)$ are the values of the European call and put options respectively, $E$ denotes the expirations price for the option, and function $\max(x,0)$ gives the large value between $x$ and 0.

The structure of this paper is organized as follows: In section 2 some basic definitions of fractional calculus and Mohand transform are given. The basic idea of the homotopy perturbation method is presented in section 3. In section 4 the problem with the solution algorithm is given. In section 5 two examples from literature are presented. A discussion of the results is given in section 6. Finally, the conclusion is drowning in section 7.

## BASIC DEFINITIONS

**Definition 2.1:** A real function $f(x), x > 0$is said to be in space $C_\mu, \mu \in R$ if there exists a real number $p \geq \mu$, such that $f(x) = x^p f_1(x)$ where $f_1(x) \in C(0,\infty)$ and it is said to be in the space $C_\mu^n$ if and only if $f^n \in C_\mu, n \in N$

**Definition 2.2** The Riemann-Liouville fractional integral operator of order $\alpha > 0$ is defined as:

$$(J^\alpha f) = \frac{1}{\Gamma(\alpha)}\int_a^x \frac{f(t)dt}{(x-t)^{1-\alpha}}, \quad (x > a, \alpha > 0)$$

For Riemann-Liouville fractional integral, one has

$$J^\alpha x^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)}$$

**Definition 2.3:** the Caputo fractional derivative of a function $f(t)$ of order $\alpha$ is defined as:

$$D_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)}\int_a^x \frac{f^n(t)dt}{(x-t)^{\alpha+1-n}}, \quad n-1 < \alpha \leq n \qquad (3)$$

Lemma

If $m-1 < \alpha \leq m, \quad m \in N, f \in C_\mu^m, \ \mu > -1$, then the following two properties hold

1. $D^\alpha[J^\alpha f(x)] = f(x)$.
2. $J^\alpha[D^\alpha f(x)] = f(x) - \sum_{k=1}^{m-1} f^k(0)\frac{x^k}{k!}$

**Definition 2.4:** The Mittag-Laffler function $E_\alpha(z)$ with $\alpha > 0$ is defined by the following series representation, valid in the whole complex plane

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n+1)} \qquad (4)$$

**Definition 2.5:** (Mohand & Mahgoub, 2017)

Consider a set $A$ defined as

$$A = \left\{ f(t): \exists\, M, \tau_1, \tau_2 > 0, |f(t)| \le M e^{\frac{|t|}{\tau_j}}, \quad if\ t \in (-1)^j \times [0,\infty) \right\} \qquad (5)$$

The Mohand transform denoted by operator $M(\cdot)$ defined by integral

$$M[f(t)] = R(u) = u^2 \int_0^\infty f(t) e^{-ut} dt, \quad t \ge 0,\ u \in [\tau_1, \tau_2] \qquad (6)$$

The variable $u$ in this transform is used to factor the variable $t$ in argument of the function $f$.

The function $f(t)$ in equation (6) is called the inverse Mohand transform of $F(u)$ and is denoted by $f(t) = M^{-1}[R(u)]$.

## Some Properties of Mohand Transform

a. Linearity property of Mohand transforms: If Mohand transform of functions $F_1(t)$ and $F_2(t)$ are $R_1(u)$ and $R_2(u)$ respectively, then Mohand transform of $[aF_1(t) + bF_2(t)]$- is given by $[aR_1(u) + bR_2(u)]$, where a, b are arbitrary constants.
b. Change of scale property: If Mohand transform of function $F(t)$ is $R(u)$ then Mohand transform of function $F(at)$ is given by a $R\left(\frac{u}{a}\right)$.
c. Convolution theorem for Mohand transforms: If Mohand transform of functions $F_1(t)$ and $F_2(t)$ are $R_1(u)$ and $R_2(u)$ respectively, then Mohand transform of their convolution $F_1(t) * F_2(t)$ is given

$$\{F_1(t) * F_2(t)\} = \left(\frac{1}{u^2}\right) M\{F_1(t)\} M\{F_1(t)\} = \left(\frac{1}{u^2}\right) R_1(u)\, R_2(u)$$

Where $F_1(t) * F_2(t)$ is defined by

$$F_1(t) * F_2(t) = \int_0^t F_1(t-x)\, F_2(x)\, dx = \int_0^t F_1(x)\, F_2(t-x) dx$$

d. Derivative theorem: Let R(u) be the Mohand transform of $M[f(t)] = R(u)$ then

$$M[f^n(t)] = u^{(n)} R(u) - \sum_{k=0}^{n-1} u^{n-k+1} f^{(k)}(0)$$

**e.** Fractional derivative theorem**:** Let $M[f(t)] = R(u)$ be the Mohand transform of a piece-wise continuous and exponential order function $f(t)$. The Mohand transform for the fractional order derivative of the function $f(t)$ under the classical Caputo fractional order derivative operator of order $\alpha > 0$ is devised as

$$M[D_t^\alpha f(t)] = u^{(\alpha)} R(u) - \sum_{k=0}^{n-1} \frac{f^{(k)}(t)}{u^{k-\alpha-1}} \qquad (7)$$

For further details and properties of Mohand transform see (Aggarwal & Chauhan, 2019; Aggarwal et al., 2018).

## The Homotopy Perturbation Method

o illustrate the basic idea of this method, we consider the following nonlinear differential equation

$$A(u) - f(r) = 0 \quad r \in \Omega \qquad (8)$$

with boundary conditions

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma \qquad (9)$$

Subject to the initial condition:

$$u^{(k)}(0) = c_k \qquad (10)$$

where $A$ is a general differential operator, $B$ is a boundary operator, $f(r)$ is a known analytic function, and $\Gamma$ is the boundary of domain $\Omega$. In general, the operator $A$ can be divided into two parts $L$ and $N$ where $L$ is a linear operator while $N$ is the nonlinear operator. Eq. (8) therefore can be written as follows:

$$L(u) + N(u) - f(r) = 0 \quad (11)$$

By the homotopy technique [24, 25] we construct a homotopy $v(r,p)\colon \Omega \times [0,1] \to R$ which satisfies

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \;\; p \in [0,1], r \in \Omega \quad (12)$$

Or

$$H(v,p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0 \quad (13)$$

where $p \in [0,1]$ is an embedding parameter, $u_0$ is an initial approximation of Eq. (8) which satisfies the boundary conditions.
From (12) and (13) we have

$$H(v,0) = L(v) - L(u_0) = 0$$
$$H(v,1) = A(v) - f(r) = 0 \quad (14)$$

The changing in the process of $p$ from zero to unity is just that of $H(r,p)$ from $u_0(r)$ to $u(r)$. In topology, this is called deformation and $L(v) - L(u_0)$, and $A(v) - f(r)$ are called homotopic.

Now, assume that the solution of equation (12) and (13) can be expressed as

$$v = v_0 + pv_1 + p^2 v_2 + \cdots\cdots \quad (15)$$

The approximate solution of Eq. (3.1) can be obtained by setting $p = 1$

$$u(x,t) = \lim_{p\to 1} v = v_0 + v_1 + v_2 + \cdots\cdots \quad (16)$$

## The Problem with the Solution Algorithm

We consider the following fractional Black-Scholes (1) of the form

$$\frac{\partial^\alpha u}{\partial t^\alpha} + \frac{\sigma x^2}{2}\frac{\partial^2 u}{\partial x^2} + r(t)x\frac{\partial u}{\partial x} - r(t)u] = 0,$$

$$(x,t) \in \mathbb{R}^+ \times (0,T), \qquad 0 < \alpha \le 1$$

Firstly, applying the Mohand transform on both sides of (1) subject to initial condition (2), we have

$$M[u(x,t)] = uu(x,0) + u^{-\alpha} M\left[\frac{\sigma x^2}{2}u_{xx} + r(t)xu_x - r(t)u\right] \quad (17)$$

By operating the inverse Mohand transform on both sides in (17), we have

$$u(x,t) = u(x,0) - M^{-1}[\, u^{-\alpha} M[\frac{\sigma x^2}{2} u_{xx} + r(t)xu_x - r(t)u]] \quad (18)$$

Now, applying the (HPM) we get

$$\sum_{n=0}^{\infty} p^n u_n\,(x,t) = u(x,0) - p\left(M^{-1}\left[u^{-\alpha} M\left[\sum_{n=0}^{\infty} p^n H_n(u)\right]\right]\right) \quad (19)$$

where

$$H_n = \frac{\sigma x^2}{2} u_{nxx} + r(t)xu_{nx} - r(t)u_n]$$

Equating the corresponding power of $p$ on both sides in equation (19), we have

$$p^0\colon u_0(x,t) = u(x,0),$$

$$p^1\colon u_1(x,t) = M^{-1}(\, u^{-\alpha} M[H_0(u)])$$

$$p^2\colon u_2(x,t) = M^{-1}(\, u^{-\alpha} M[H_1(u)]) \quad (20)$$

$$\vdots$$

$$p^n: u_n(x,t) = M^{-1}( u^{-\alpha} M[H_{n-1}(u)])$$

So, that the solution $u(x,t)$ of the problem is given by

$$u(x,t) = \lim_{n\to\infty} \sum_{n=0}^{n} u_n(x,t)$$

## APPLICATIONS

In this section, we discuss the implementation of the proposed method.

Example 5.1. We consider the following fractional Black-Scholes option pricing equation as follows:

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\sigma x^2}{2}\frac{\partial^2 u}{\partial x^2} + (k-1)\frac{\partial u}{\partial x} - ku], \quad 0 < \alpha \le 1 \quad (21)$$

subject to initial condition

$$u(x,0) = \max(e^x - 1, 0) \quad (22)$$

Applying the Mohand transform on both sides of (21) subject to initial condition (22), we have

$$M[u(x,t)] = uu(x,0) + u^{-\alpha} M[u_{xx} + (k-1)u_x - ku] \quad (23)$$

Operating the inverse Mohand transform on both sides of (23), we have

$$u(x,t) = \max(e^x - 1, 0) + M^{-1}[ u^{-\alpha} M[u_{xx} + (k-1)u_x - ku]]$$

Now, applying (HPM)

$$\sum_{n=0}^{\infty} p^n u_n\,(x,t) = \max(e^x - 1, 0) + p\left(M^{-1}\left[u^{-\alpha} M\left[\sum_{n=0}^{\infty} p^n H_n(u)\right]\right]\right) \quad (24)$$

where

$$H_n = u_{nxx} + (k-1)u_{nx} - rku_n]$$

Equating the corresponding power of $p$ on both sides in equation (24), we have

$$p^0: u_0(x,t) = \max(e^x - 1, 0),$$

$$p^1: u_1(x,t) = M^{-1}( u^{-\alpha} M[H_0(u)])$$

$$= -\max(e^x, 0)\frac{(-kt^\alpha)}{\Gamma(\alpha+1)} + \max(e^x - 1, 0)\frac{(-kt^\alpha)}{\Gamma(\alpha+1)}$$

$$p^2: u_2(x,t) = M^{-1}( u^{-\alpha} M[H_1(u)])$$

$$= -\max(e^x, 0)\frac{(-kt^\alpha)^2}{\Gamma(2\alpha+1)}$$

$$+ \max(e^x - 1, 0)\frac{(-kt^\alpha)^2}{\Gamma(2\alpha+1)}$$

$$\vdots$$

$$p^n: u_n(x,t) = M^{-1}( u^{-\alpha} M[H_{n-1}(u)])$$

$$= -\max(e^x, 0)\frac{(-kt^\alpha)^n}{\Gamma(n\alpha+1)}$$

$$+\max(e^x - 1, 0)\frac{(-kt^\alpha)^n}{\Gamma(n\alpha+1)}$$

So that solution $u(x,t)$ of the problem is given by

$$u(x,t) = \lim_{n\to\infty} \sum_{n=0}^{n} u_n(x,t)$$

$$= \max(e^x - 1, 0)E_\alpha(-kt^\alpha)$$

$$+\max(e^x, 0)\big(1 - E_\alpha(kt^\alpha)\big)$$

where $E_\alpha(-kt^\alpha)$ is a Mittag-Leffler function in one parameter.

For the special case $\alpha = 1$, we get

$$u(x,t) = \max(e^x - 1, 0)e^{-kt} + \max(e^x, 0)(1 - e^{-kt})$$

Which is an exact solution of Black-Scholes equation (21) for $\alpha = 1$

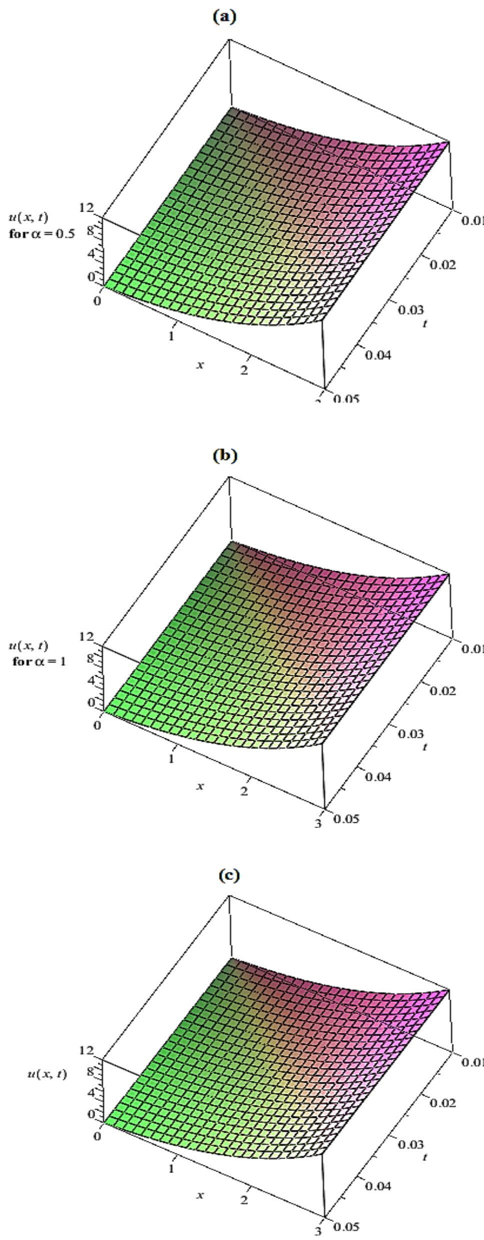


**Figure (1).** (a) approximate solution $\alpha = 0.5$, (b) approximate solution $\alpha = 1$, and (c) exact solution for equation (21)

Example 5.2. We consider the following fractional Black-Scholes option pricing equation as follows:

$$\frac{\partial^\alpha u}{\partial t^\alpha} + 0.08(2 + \sin x)^2 x^2 \frac{\partial^2 u}{\partial x^2} + 0.06x\frac{\partial u}{\partial x} - 0.06u = 0 \quad ,0 < \alpha \le 1 \quad (25)$$

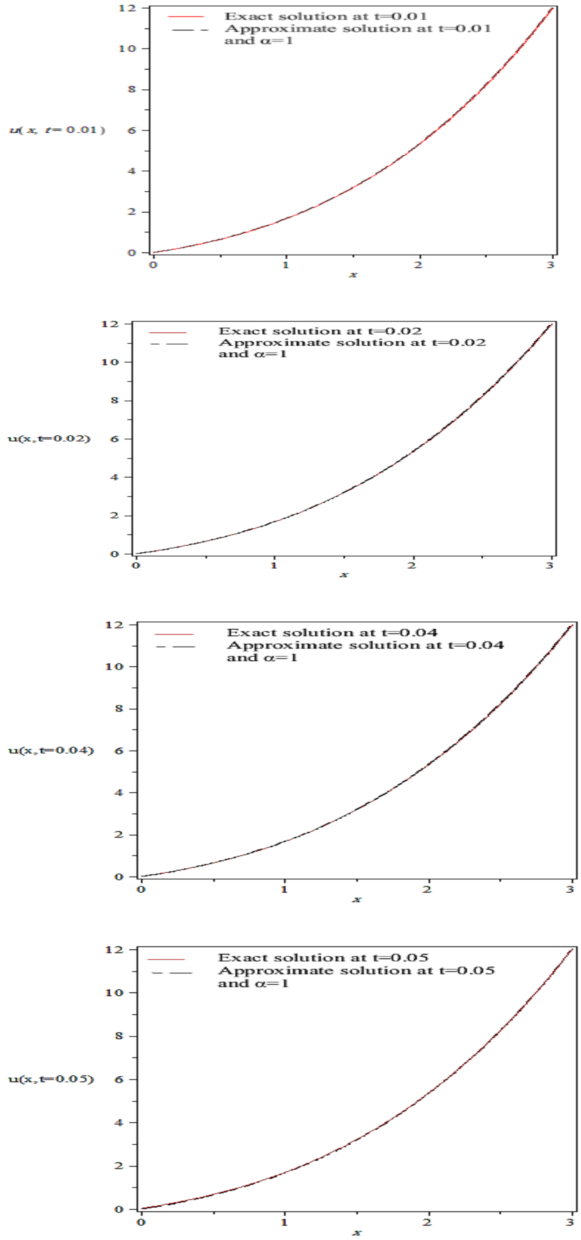


**Figure (2).** Comparison of approximate solution with exact solution at different times $t = 0.01, 0.02, 0.04, 0.05$ for Eq.(21) with initial condition (22) with $\alpha = 1$.

subject to the initial condition

$$u(x,0) = \max(x - 25e^{-0.06}, 0) \quad (26)$$

Firstly, applying the Mohand transform on both sides of (25), subject to initial condition (26), we have

$$M[u(x,t)] = uu(x,0) - u^{-\alpha} M[0.08 \, (2+\sin x)^2 x^2 u_{xx} + 0.06 x u_x - 0.06u] \quad (27)$$

Operating the inverse Mohand transform on both sides of (27), we have

$$u(x,t) = \max(x - 25e^{-0.06}, 0) - M^{-1}[\, u^{-\alpha} M[0.08 \, (2+\sin x)^2 x^2 u_{xx} + 0.06 x u_x - 0.06u$$

Now, applying (HPM)

$$\sum_{n=0}^{\infty} p^n u_n\,(x,t) = \max(x - 25e^{-0.06}, 0) - p\left( M^{-1}\left[ u^{-\alpha} M\left[ \sum_{n=0}^{\infty} p^n H_n(u) \right]\right]\right) \quad (28)$$

where

$$H_n = 0.08 \, (2+\sin x)^2 x^2 u_{xx} + 0.06 x u_x - 0.06u$$

Equating the corresponding power of $p$ on both sides in equation (28), we have

$$p^0 : u_0(x,t) = \max(x - 25e^{-0.06}, 0),$$

$$p^1 : u_1(x,t) = M^{-1}(\, u^{-\alpha} M[H_0(u)]) = -x\frac{(-0.06t^{\alpha})}{\Gamma(\alpha+1)} + \max(x - 25e^{-0.06}, 0)\frac{(-0.06t^{\alpha})}{\Gamma(\alpha+1)}$$

$$p^2 : u_2(x,t) = M^{-1}(\, u^{-\alpha} M[H_1(u)]) = -x\frac{(-0.06t^{\alpha})^2}{\Gamma(2\alpha+1)} + \max(x - 25e^{-0.06}, 0)\frac{(-0.06t^{\alpha})^2}{\Gamma(2\alpha+1)}$$

$$\vdots$$

$$p^n : u_n(x,t) = M^{-1}(\, u^{-\alpha} M[H_{n-1}(u)]) = -x\frac{(-0.06t^{\alpha})^n}{\Gamma(n\alpha+1)} + \max(x - 25e^{-0.06}, 0)\frac{(-0.06t^{\alpha})^n}{\Gamma(n\alpha+1)}$$

So that solution $u(x,t)$ of the problem is given by

$$u(x,t) = \lim_{n\to\infty} \sum_{n=0}^{n} u_n(x,t) = x(1 - E_{\alpha}(-0.06t^{\alpha}) + \max(x - 25e^{-0.06}, 0) E_{\alpha}(-0.06t^{\alpha})$$

where $E_{\alpha}(-kt^{\alpha})$ is a Mittag-Leffler function in one parameter.

For the special case $\alpha = 1$, we get

$$u(x,t) = x(1 - e^{0.06t}, 0) + \max(x - 25, 0)e^{-0.06}$$

Which is an exact solution of Black-Scholes equation (25) for $\alpha = 1$

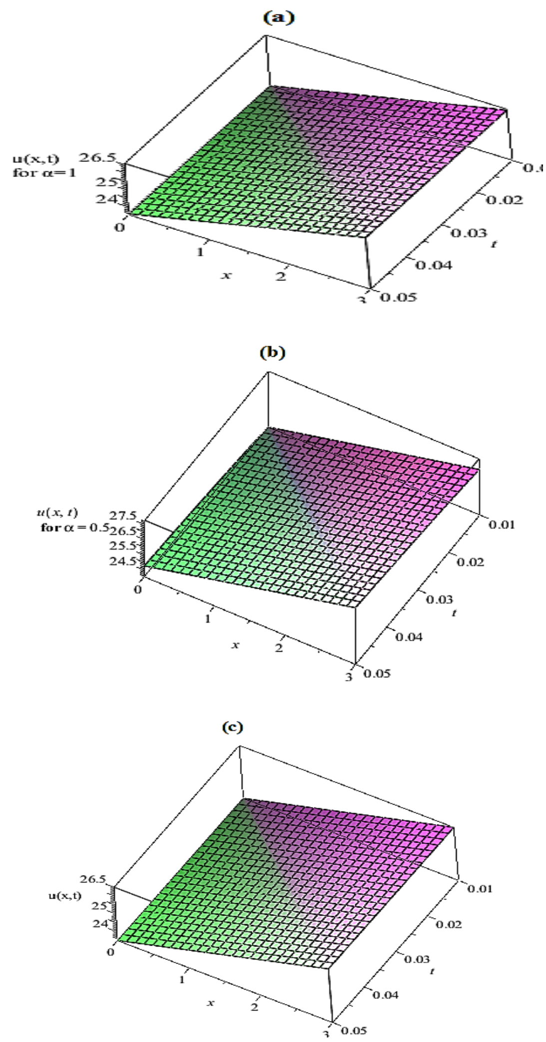


**Figure (3).** (a) approximate solution $\alpha = 0.5$, (b) approximate solution $\alpha = 1$, and (c) exact solution for equation (25)-(26)

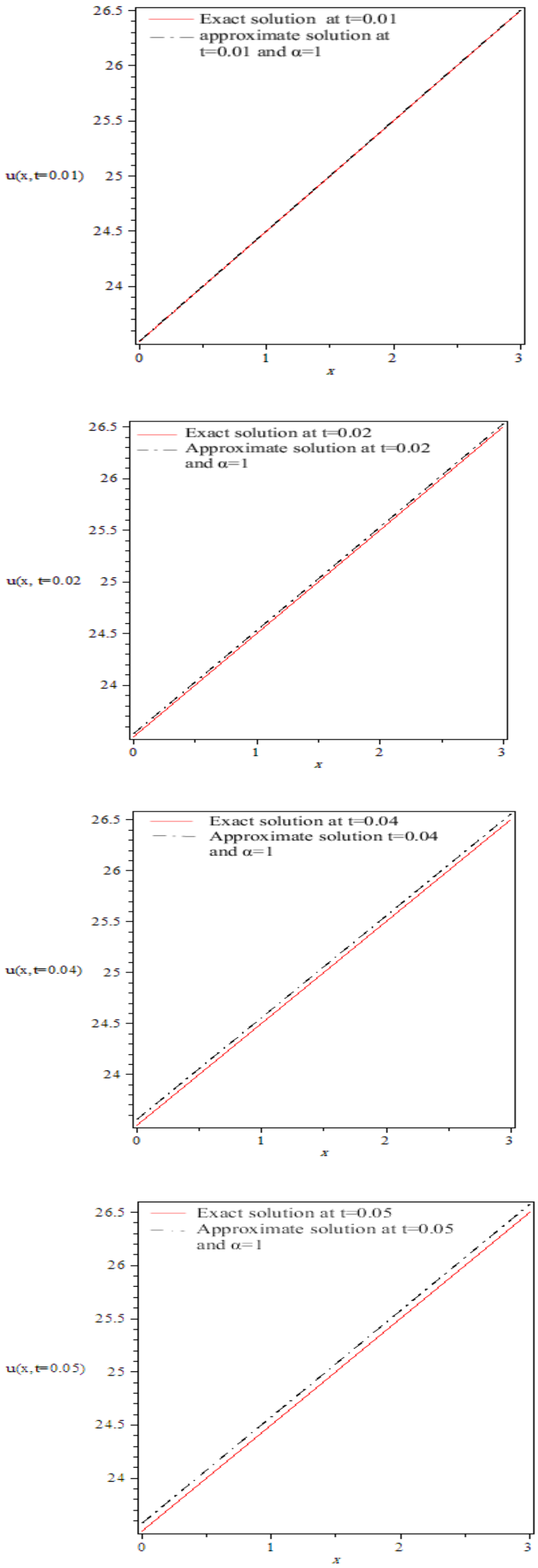


**Figure (4).** Comparison of approximate solution with exact solution at different times $t = 0.01,0.02,0.04,0.05$ for Eq. (25) with the initial condition (26), with $\alpha = 1$.

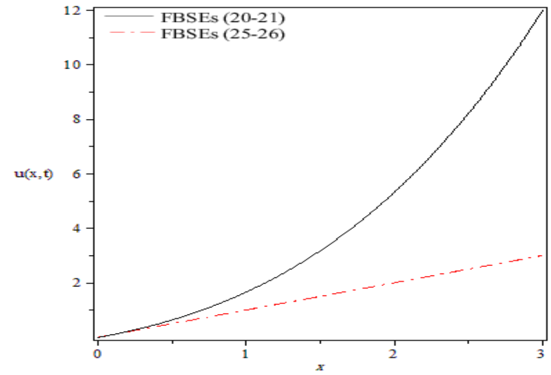


**Figure (5).** Comparison of the solution for equation (21)-(22) and equation (25)-(26) with $\alpha = 0.5$ at time $t = 0.01$

## DISCUSSION

Figures (1) (a and b) and (3) (a and b) display the change in the solution behaviors when $\alpha = 1$ and $0.5$ respectively. In both subfigure, we see that the approximate solution is very close to the exact solution subfigure (c) of (1) and (3).

Further, Figures (2) and (4) show the comparison of the exact solution with the (HPMTM) solution at $\alpha = 1$ and $t = 0.01, 0.02, 0.04$ and $0.05$ for equations (21)-(22) and (25)-(26) respectively. It is clear that the (HPMTM) solutions seem to coincide with the actual solution. Finally, Figure (5) provides a comparison of (21)-(22) and (25)-(26) with $\alpha = 0.5$ and $\alpha = 0.01$, where it can be concluded that the difference between the solution of fractional Black-Scholes (21)-(22) and the solution of fractional Black-Scholes (25)-(26) is due to the difference in $\sigma$ in both equations.

## CONCLUSION

In this paper, the homotopy perturbation method was coupled with Mohand transforms (HPMMT) and successfully applied to get the approximate analytical solution of the fractional Black-Scholes option pricing equation in terms of convergent series with easily computable components. The results show that this method is a powerful tool for obtaining exact and approximate analytical solutions of fractional Black-Scholes European option equations.

**Table (1).** Mohand transform of some basic mathematical functions

| S.N. | $F(t)$ | $M[F(t)] = R(u)$ |
|---|---|---|
| 1 | 1 | $u$ |
| 2 | $t$ | 1 |
| 3 | $t^2$ | $\frac{2!}{u}$ |
| 4 | $t^n, n \in \mathbb{N}$ | $\frac{n!}{u^{n-1}}$ |
| 5 | $t^n$ | $\frac{\Gamma(n+1)}{u^{n-1}}$ |
| 6 | $e^{at}$ | $\frac{u^2}{(u-a)}$ |
| 7 | $\sin at$ | $\frac{au^2}{(u^2+a^2)}$ |
| 8 | $\cos at$ | $\frac{u^3}{(u^2+a^2)}$ |
| 9 | $\sinh at$ | $\frac{au^2}{(u^2-a^2)}$ |
| 10 | $\cosh at$ | $\frac{u^3}{(u^2-a^2)}$ |

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