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On the Mohand Transform and Ordinary Differential Equations with Variable Coefficients

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Abstract: The Mohand transform is a new integral transform introduced by Mohand M. Abdelrahim Mahgoub to facilitate the solution of differential and integral equations. In this article, a new integral transform, namely Mohand transform was applied to solve ordinary differential equations with variable coefficients by using the modified version of Laplace and Sumudu transforms.

Keywords: Mohand Transform; Differential Equations.

INTRODUCTION

Integral transforms play an important role in many fields of science. In the literature, an integral transform is widely used in physics, astronomy, optics, and engineering mathematics.

The term "Differential Equation" was proposed in 1676 by Leibniz. The first studies of these equations were carried out in the late 17th century. Differential equations are powerful tools in the study of many problems in science and technology (Khan et al., 2018).

Recently, Mohand M. Mahgoub introduces a new integral transform named the "Mohand Transform", and it has further applied to the solution of ordinary and partial differential equations. The purpose of this paper is to solve differential equations with variable coefficients using Mohand Transform.

DEFINITIONS AND STANDARD RESULTS

Definition 2.1 (Mohand Transform) (Mohand & Mahgoub, 2017):

A new transform called the Mohand Transform is defined for a function of exponential order. We consider functions in the set A defined by

$$A = \left\{ \begin{array}{l} f(t) : \exists M, K_1, K_2 > 0, |f(t)| < M e^{\frac{|t|}{k_j}}, \\ if\ t \in (-1)^j \times [0, \infty) \end{array} \right\}$$

For a given function in the set A, The constant M must be a finite number k_1, k_2 may be finite or infinite.

The Mohand Transform denoted by operator $\boldsymbol{M}(\cdot)$ defined by the integral:

$$M[f(t)] = R(v) = v^2 \int_0^\infty f(t) e^{-vt} dt \quad , t \geq 0 ,$$

$K_1 \leq v \leq K_2$

1.1 Some Properties of Mohand Transform(Aggarwal & Chauhan, 2019)

a) Linearity Property :

if $M[f_1(t)] = R_1(v)$, $M[f_2(t)] = R_2(v)$ then

$$M[af_1(t) + bf_2(t)] = aR_1(v) + bR_2(v)$$

where a, b are arbitrary constants.

b) Change of Scale Property:

if $M[f(t)] = R(v)$ then

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$$M\left[f\left(at\right)\right]=aR\left(\frac{v}{a}\right)$$

c) Shifting Property:

if $M\left[f\left(t\right)\right]=R\left(v\right)$ then

$$M\left[e^{at}f\left(t\right)\right]=\left[\frac{v^2}{\left(v-a\right)^2}\right]R\left(v-a\right)$$

d) Convolution Theorem:

if $M\left[f_1\left(t\right)\right]=R_1\left(v\right)$, $M\left[f_2\left(t\right)\right]=R_2\left(v\right)$ then

$$M\left[f_1\left(t\right)*f_2\left(t\right)\right]=\frac{1}{v^2}R_1\left(v\right)\cdot R_2\left(v\right)$$

where $f_1\left(t\right)*f_2\left(t\right)$ is defined by

$$f_1\left(t\right)*f_2\left(t\right)=\int_0^t f_1\left(t-x\right)f_2\left(x\right)dx$$

$$=\int_0^t f_1\left(x\right)f_2\left(t-x\right)dx$$

e) Mohand Transform of the Integral of a Function:

$f\left(t\right)$: if $M\left[f\left(t\right)\right]=R\left(v\right)$ then

$$M\left[\int_o^t f\left(t\right)dt\right]=\frac{1}{v}R\left(v\right)$$

f) Mohand Transform of the Derivatives:

if $M\left[f\left(t\right)\right]=R\left(v\right)$, then

i) $M\left[f'\left(t\right)\right]=vR\left(v\right)-v^2f\left(0\right)$

ii) $M\left[f''\left(t\right)\right]=v^2R\left(v\right)-v^3f\left(0\right)-v^2f'\left(0\right)$

iii) $M\left[tf\left(t\right)\right]=\left[\frac{2}{v}-\frac{d}{dv}\right]R\left(v\right)$

iv) $M\left[tf'\left(t\right)\right]=2R\left(v\right)-2vf\left(0\right)-\frac{d}{dv}\left[vR\left(v\right)-v^2f\left(0\right)\right]$

v) $M\left[tf''\left(t\right)\right]=2vR\left(v\right)-2v^2f\left(0\right)-2\times \times vf'\left(0\right)-\frac{d}{dv}\left[v^2R\left(v\right)-v^3f\left(0\right)-v^2f'\left(0\right)\right]$

Notice that (i), (ii) are proved in (Mohand & Mahgoub, 2017) and (iii) are proved in (Aggarwal & Chauhan, 2019)

And from (iii), one can deduce that

$$M\left[tf'\left(t\right)\right]=\left[\frac{2}{v}-\frac{d}{dv}\right]M\left[f'\left(t\right)\right]$$

$$=\left[\frac{2}{v}-\frac{d}{dv}\right]\left[vR\left(v\right)-v^2f\left(0\right)\right]$$

$$=2R\left(v\right)-2vf\left(0\right)-\frac{d}{dv}\left[vR\left(v\right)-v^2f\left(0\right)\right]$$

And in a similar way, one can deduce that

$$M\left[tf''\left(t\right)\right]=\left[\frac{2}{v}-\frac{d}{dv}\right]M\left[f''\left(t\right)\right]$$

$$=\left[\frac{2}{v}-\frac{d}{dv}\right]\left[v^2R\left(v\right)-v^3f\left(0\right)-v^2f'\left(0\right)\right]$$

$$=2vR\left(v\right)-2v^2f\left(0\right)-2vf'\left(0\right)-\frac{d}{dv}\left[v^2R\left(v\right)-v^3f\left(0\right)-v^2f'\left(0\right)\right]$$

g) Theorem 2.1: if $M\left[f\left(t\right)\right]=R\left(v\right)$ then

$$\lim_{v\to\infty}\left[\frac{1}{v^2}R\left(v\right)\right]=0$$

Proof:

$$M\left[f\left(t\right)\right]=R\left(v\right)=v^2\int_0^\infty f\left(t\right)e^{-vt}dt$$

$$\Rightarrow\ \frac{1}{v^2}R\left(v\right)=\int_0^\infty e^{-vt}f\left(t\right)dt$$

$$\Rightarrow\ \lim_{v\to\infty}\left[\frac{1}{v^2}R\left(v\right)\right]=\lim_{v\to\infty}\int_0^\infty e^{-vt}f\left(t\right)dt$$

$$=\int_0^\infty \lim_{v\to\infty}e^{-vt}f\left(t\right)dt$$

$$=0$$

$$\therefore\ \lim_{v\to\infty}\left[\frac{1}{v^2}R\left(v\right)\right]=0$$

Mohand Transform of Some Functions (Aggarwal et al., 2018)

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S.N	$f(t)$	$M[f(t)] = R(v)$
1.	1	v
2.	t	1
3.	t^2	$\frac{2!}{v}$
4.	$t^n, \quad n \in \mathbb{N}$	$\frac{n!}{v^{n-1}}$
5.	e^{at}	$\frac{v^2}{v-a}$
6.	$\sin at$	$\frac{av^2}{v^2+a^2}$
7.	$\cos at$	$\frac{v^3}{v^2+a^2}$
8.	$\sinh at$	$\frac{av^2}{v^2-a^2}$
9.	$\cosh at$	$\frac{v^3}{v^2-a^2}$
10.	$J_0(t)$	$\frac{v^2}{\sqrt{1+v^2}}$
11.	$J_0(at)$	$\frac{v^2}{\sqrt{a^2+v^2}}$
12.	$J_1(t)$	$v^2 - \frac{v^3}{\sqrt{1+v^2}}$

APPLICATIONS

Example 3.1 (Khan et al., 2018)

Solve the differential equation:

$$y'' + t\,y' - y = 0$$

with the initial condition,

$$y(0) = 0, y'(0) = 1$$

Solution: Taking Mohand transform to give the following equation

$$v^2R(v) - v^3f(0) - v^2f'(0) + 2R(v) - 2v\,f(0)$$

$$-\frac{d}{dv}\left[vR(v) - v^2f(0)\right] - R(v) = 0$$

$$v^2R(v) - v^2 + R(v) - v\,R'(v) - R(v) = 0$$

$$R'(v) - vR(v) = -v$$

which is a linear differential equation. Its solution is

$$R(v) = 1 + c\,e^{\frac{v^2}{2}}$$

$$\Rightarrow \quad R(v) = 1 \qquad (\text{by Theorem 2.1} \;\; c = 0)$$

By using inverse Mohand Transform, we get

$$y(t) = t$$

Example 3.2 (Nagle et al., 2014)

Consider the ordinary differential equation:

$$y'' + 2t\,y' - 4y = 1$$

with the initial condition,

$$y(0) = 0, y'(0) = 0$$

Solution: Taking Mohand transform to given equation

$$v^2R(v) - v^3f(0) - v^2f'(0) + 4R(v) - 4v\,f(0)$$

$$-2\frac{d}{dv}\left[vR(v) - v^2f(0)\right] - 4R(v) = v$$

$$v^2R(v) - 2v\,R'(v) - 2R(v) = v$$

$$R'(v) - \left[\frac{v^2-2}{2v}\right]R(v) = \frac{-1}{2}$$

Which is a linear differential equation. Its solution is

$$R(v) = \frac{1}{v} + \frac{1}{v}c\,e^{\frac{v^2}{4}}$$

$$\Rightarrow \quad R(v) = \frac{1}{v} \qquad (\text{by Theorem 2.1} \;\; c = 0)$$

$$\Rightarrow \quad R(v) = \frac{1}{2!}\left[\frac{2!}{v}\right]$$

By using inverse Mohand Transform, we get

$$y(t) = \frac{1}{2}t^2$$

Example 3.3 (Raisinghania, 2009)

Solve the differential equation:

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$$t\,y'' - y' = t^2$$

with the initial condition ,

$$y(0) = 0, y'(0) = 0$$

Solution: Taking Mohand transform to given equation

$$2v\,R(v) - 2v^2 f(0) - 2v f'(0) - \frac{d}{dv}\Big[v^2 R(v) -$$

$$v^3 f(0) - v^2 f'(0)\Big] - vR(v) + v^2 f(0) = \frac{2}{v}$$

$$v\,R(v) - 2v\,R(v) - v^2 R'(v) = \frac{2}{v}$$

$$R'(v) + \frac{1}{v}R(v) = \frac{-2}{v^3}$$

Which is a linear differential equation. Its solution is

$$R(v) = \frac{2}{v^2} + \frac{c}{v}$$

By using inverse Mohand Transform, we get

$$y(t) = \frac{1}{3}t^3 + \frac{c}{2}t^2$$

Example 3.4 (Raisinghania, 2009)
Solve the differential equation:

$$t\;y'' + y' + 4t\,y = 0$$

with the initial condition,

$$y(0) = 3, y'(0) = 0$$

Solution: Taking Mohand transform to given equation

$$2v\,R(v) - 2v^2 f(0) - 2v f'(0) - \frac{d}{dv}\Big[v^2 R(v) -$$

$$v^3 f(0) - v^2 f'(0)\Big] + vR(v) - v^2 f(0) +$$

$$4\left[\frac{2}{v} - \frac{d}{dv}\right]R(v) = 0$$

$$2v\,R(v) - 6v^2 - 2v\,R(v) - v^2 R'(v) + 9v^2 +$$

$$v\,R(v) - 3v^2 + \frac{8}{v}R(v) - 4R'(v) = 0$$

$$\left(v^2+4\right)R'(v) - \left(v + \frac{8}{v}\right)R(v) = 0$$

$$\Rightarrow \quad R'(v) = \left[\frac{v^2+8}{v\left(v^2+4\right)}\right]R(v)$$

$$\Rightarrow \quad \frac{d\left[R(v)\right]}{R(v)} = \frac{v^2+8}{v\left(v^2+4\right)} = \frac{\left(v^2+4\right)+4}{v\left(v^2+4\right)}$$

$$\Rightarrow \quad \frac{d\left[R(v)\right]}{R(v)} = \frac{2}{v} - \frac{v}{\left(v^2+4\right)}$$

$$\Rightarrow \quad R(v) = c\;\frac{v^2}{\sqrt{v^2+4}}$$

By using inverse Mohand Transform, we get

$$y(t) = c\;J_0(2t)$$

By using initial condition $y(0) = 3$

Since $J_0(0) = 1$, we get $c = 3$

$$\therefore y(t) = 3\,J_0(2t)$$

CONCLUSION

In this paper, we apply a new integral transform "Mohand Transform" to solve some ordinary differential equations with variable coefficients, and all solutions are satisfied by putting them back in the corresponding equations. The result reveals that the proposed method is very efficient, simple, and can be applied to linear differential equations.

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حول تحويل مهند والمعادلات التفاضلية العادية ذات المعاملات المتغيرة

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المستخلص: تحويل مهند هو تحويل تكاملي جديد قدمه مهند م. عبد الرحيم محجوب وذلك لتسهيل حل المعادلات التفاضلية والمعادلات التكاملية. في هذه الورقة سوف يطبق تحويل تكاملي جديد ألا وهو تحويل مهند لحل معدلات تفاضلية عادية بمعاملات متغيرة باستخدام صيغ معدلة من تحويل لابلاس و تحويل سيوميودو.

الكلمات المفتاحية: تحويل مهند، معادلات تفاضلية.

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