

# **Optical Soliton Solutions to Gerdjikov-Ivanov Equation Without Four-Wave Mixing Terms in Birefringent Fibers by Extended Trial Function Scheme**

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**Abstract:** Without four-wave mixing terms in birefringent fibers, the extended trial function scheme was used to obtain optical soliton solutions for the coupled system corresponding to the Gerdjikov-Ivanov equation. The procedure reveals singular soliton solutions, bright soliton solutions, and highly important solutions in terms of Jacobi's elliptic function. And in the limiting case of the modulus of ellipticity, singular and singular-periodic soliton solutions, along with their respective existence criteria.

**Keywords:** Birefringent Fibers, The Coupled Gerdjikov-Ivanov Model Without Four-Wave Mixing Terms, Extended Trial Function Scheme, Optical Solutions.

# INTRODUCTION

The Gerdjikov-Ivanov (GI) model without four-wave mixing terms (FWM) is one of the varieties of models that study the dynamics of optical soliton propagation for transmission transcontinental technology, the and transoceanic distances, optical fibers, data and the telecommunications transmission, industry. This model has been studied for polarization-preserving fibers along with strategic algorithms such as modified simple equation scheme, the csch method, the extended  $\tanh - \coth$  method,  $\frac{G'}{G^2}$ -expansion method, sine-cosine method, trial, and the extended trial equation method, trial equation integration architecture. extended Kudryashov's method, and the  $\exp(-(\phi))$ expansion method (Arshed, 2018; Arshed et al., 2018; Biswas, Ekici, Sonmezoglu, Majid, et al., 2018; Biswas, Ekici, Sonmezoglu, Triki, et al., 2018; Biswas, Yildirim, Yasar, Triki, et al., 2018a, 2018b; Biswas, Yıldırım, et al., 2018; Ekici et al.,2017; Jawad et al.,2018; Kadkhoda, N.; Jafari, 2016; Yildirim, 2019d, 2019a, 2019b, 2019c) and the extended simplest

equation method (Hassan & Altwaty, 2020). Although there are many advancements, the solitons were taken into account only along one model component. The extended trial function scheme has been applied to the coupled GI model without FWM given in two-component forms in birefringent fibers which gives rise to improving the model further. The strategy of the method reveals singular and bright soliton solutions. Furthermore, highly important solutions in terms of Jacobi's elliptic function, and in the limiting case of the modulus of ellipticity. singular and singular-periodic soliton solutions have been gained and listed with their respective existence criteria.

## **GOVERNING MODEL**

The (GI) equation (Arshed, 2018; Arshed et al., 2018; Biswas, Ekici, Sonmezoglu, Majid, et al., 2018; Biswas, Ekici, Sonmezoglu, Triki, et al., 2018; Biswas, Yildirim, Yasar, Triki, et al., 2018b, 2018a; Biswas, Yıldırım, et al., 2018; Jawad et al., 2018; Yildirim, 2019d, 2019b, 2019c) is represented as

$$i\psi_t + a\psi_{xx} + b|\psi|^4\psi + ic\psi^2\psi_x^* = 0$$
 (1)

The first term is referred to as the temporal evolution of pulses when the existence of group velocity dispersion is supplied by the coefficient of a in this quite important governing model. The complex-valued function  $\psi(x, t)$  is referred to as the wave profile. The coefficient of b is named as the nonlinear term that signifies quintic nonlinearity. Once and for all the existence of a form of dispersive phenomenon is ensured with the coefficient of c.

The GI model without FWM in birefringent fibers (Yildirim, 2019) is described by

$$\begin{split} &i\psi_t + a_1\psi_{xx} + (b_1|\psi|^4 + c_1|\psi|^2|\phi|^2 + d_1|\phi|^4)\psi + \\ &i(\beta_1\psi^2 + \gamma_1\phi^2)\psi_x^* = 0, \\ &i\phi_t + a_2\phi_{xx} + (b_2|\phi|^4 + c_2|\phi|^2|\psi|^2 + d_2|\psi|^4)\phi + \\ &i(\beta_2\phi^2 + \gamma_2\psi^2)\phi_x^* = 0. \end{split}$$

The coefficients of  $a_j$  correspond to group velocity dispersion when the coefficients of  $b_j$  stem from self-phase modulation in this coupled GI system. Once and for all, the coefficients of  $c_j$  as well as  $d_j$  correspond to cross-phase modulation, whilst the coefficients of  $\beta_j$ ,  $\gamma_j$  account for other forms of dispersive phenomenon along with j = 1,2.

#### MATHEMATICAL PRELIMINARIES

The	starting	hypothesis	for	solving	g the
considered coupled system is given by					
$\psi(x,t)$	$z = w_1(\zeta(x))$	$(t))e^{i heta(x,t)},$			(3)
$\phi(x,t)$	$w_2(\zeta(x)) = w_2(\zeta(x))$	$(t))e^{i heta(x,t)},$			(4)
where $w_i$ represent the amplitude component of					
the s	soliton an	d $\theta_i$ for $j =$	= 1,2	is the	phase
component of the soliton that is described as					
-					( <b>-</b> )

$$\zeta(x,t) = k_1 x - vt,$$
(5)  
 $\theta(x,t) = -k_2 x + \mu t + k_3.$ 
(6)

Here, v is the velocity of the soliton,  $k_2$  is the frequency of the solitons in each of the two components while w is the soliton wave number and  $k_3$  is the phase constant. By putting (4) and (5) into (2) we get:  $-(\mu + a_1k_2^2)w_1 + a_1k_1^2w_1'' + b_1w_1^5 + c_1w_1^3w_2^2 +$ 

 $-(\mu + a_1 \kappa_2) w_1 + a_1 \kappa_1 w_1 + b_1 w_1 + c_1 w_1 w_2 + d_1 w_1 w_2^4 - k_2 \beta_1 w_1^3 - k_2 \gamma_1 w_2^2 w_1 + i(-\nu - 2a_1 k_1 k_2 + k_1 \beta_1 w_1^2 + k_1 \gamma_1 w_2^2) w_1' = 0,$ (7)

 $-(\mu + a_2k_2^2)w_2 + a_2k_1^2w_2'' + b_2w_2^5 + c_2w_2^3w_1^2 + d_2w_1^4w_2 - k_2\beta_2w_2^3 - k_2\gamma_2w_1^2w_2 + i(-\nu - 2a_2k_1k_2 + k_1\beta_2w_2^2 + k_1\gamma_2w_1^2)w_2' = 0.$ (8)

Equation (7) and (8) can be gathered as

$$-(\mu + a_j k_2^2) w_j + a_j k_1^2 w_j'' + b_j w_j^5 + c_j w_j^3 w_l^2 + d_j w_j w_l^4 - k_2 \beta_j w_j^3 - k_2 \gamma_j w_l^2 w_j + i(-\nu - 2a_j k_1 k_2 + k_1 \beta_j w_j^2 + k_1 \gamma_j w_l^2) w_j' = 0,$$
(9)

where j = 1,2 and l = 3 - j, using the balancing principle we get  $w_j = w_l$ 

$$-(\mu + a_j k_2^2) w_j + a_j k_1^2 w_j'' + (b_j + c_j + d_j) w_j^5 - k_2 (\beta_j + \gamma_j) w_j^3 + i (-\nu - 2a_j k_1 k_2 + k_1 (\beta_j + \gamma_j) w_j^2) w_j' = 0,$$
(10)

Splitting into real and imaginary parts we get:

$$-(\mu + a_j k_2^2) w_j + a_j k_1^2 w_j'' + (b_j + c_j + d_j) w_j^5 - k_2 (\beta_j + \gamma_j) w_j^3 = 0,$$
(11)

$$-\nu - 2a_jk_1k_2 + k_1(\beta_j + \gamma_j)w_j^2 = 0.$$
 (12)

Equation (12) presents the velocity of the soliton solution, balancing w'' with  $w^5$  in equation (11) gives  $N = \frac{1}{2}$ , since N is not real, we set  $w_j = \sqrt{\varphi_j}$ . Substituting into (11) and multiplying by  $4\varphi_j\sqrt{\varphi_j}$  we get  $\sigma_{(1,j)}\varphi_j^2 + \sigma_{(2,j)}\varphi_j\varphi_j'' + \sigma_{(3,j)}(\varphi_j')^2 + \sigma_{(4,j)}\varphi_j^4 + \frac{1}{2}$ 

 $\sigma_{(5,j)}\varphi_j^3 = 0, \qquad (13)$ where  $\sigma_{(1,j)} = -4(\mu + a_jk_2^2), \sigma_{(2,j)} = 2a_jk_1^2, \sigma_{(3,j)} = -a_jk_1^2, \sigma_{(4,j)} = 4(b_j + c_j + d_j), \sigma_{(5,j)} = -4k_2(\beta_j + \gamma_j).$ Polynomiae  $\alpha_i \alpha_i''$  with  $\alpha_i^4$  gives N = 1

Balancing  $\varphi_j \varphi_j''$  with  $\varphi^4$  gives N = 1

### EXTENDED TRIAL EQUATION SCHEME

The traveling wave solution with extended trial function scheme is:

$$\varphi_j = \sum_{i=0}^N A_{i,j} u^i, \quad j = 1,2,$$
 (14)

where

$$(u')^{2} = \Gamma(u) = \frac{\Theta(u)}{\Upsilon(u)} = \frac{\sum_{i=0}^{\tau} \lambda_{i} u^{i}}{\sum_{i=0}^{\rho} \chi_{i} u^{i}}$$
(15)

© 2021 The Author(s). This open access article is distributed under a CC BY-NC 4.0 license. ISSN: online 2617-2186 print 2617-2178 where  $\lambda_i$ ,  $\chi_i$ ,  $A_{i,j}$  are constants and  $\lambda_{\tau}$ ,  $\chi_{\rho}$ ,  $A_{N,j}$  are non-zero. Equation (15) can be formulated as

$$\pm(\zeta-\zeta_0) = \int \frac{du}{\sqrt{\Gamma(u)}} = \int \sqrt{\frac{\Upsilon(u)}{\Theta(u)}} du, \qquad (16)$$

The balancing principle applied to (13) implies

$$\tau = \rho + 2N + 2, \tag{17}$$

Since N = 1 and setting  $\rho = 0$ , we get  $\tau = 4$  consequently, from (14) we have

$$\varphi_j = A_{0,j} + A_{1,j}u, \tag{18}$$

$$(\varphi_j')^2 = \frac{(\pi_{1,j}) - \mathcal{L}_{l=0} - \pi_l \alpha}{\chi_0},$$
 (19)

$$\varphi_j^{\prime\prime} = \frac{(A_{1,j}) \sum_{i=0}^4 i \lambda_i u^{i-1}}{2\chi_0},\tag{20}$$

where  $\lambda_4 \neq 0$  and  $\chi_0 \neq 0$ . Substituting Eqs. (18) – (20) into Eq. (13), we obtain a system of algebraic equations. Solving the system, we get

$$\begin{split} \lambda_{0} &= \lambda_{0}, \ \lambda_{1} = \lambda_{1}, \ A_{0,j} = A_{0,j}, \ A_{1,j} = A_{1,j}, \ \chi_{0} = \chi_{0}, \\ \lambda_{2} &= \\ \frac{A_{0,j}^{4}\chi_{0}(4b_{j}-2+4c_{j}+4d_{j})+2A_{0,j}^{3}k_{2}\chi_{0}(\beta_{j}+\gamma_{j})+A_{1,j}a_{j}k_{1}^{2}(A_{0,j}\lambda_{1}-A_{1,j}\lambda_{0})}{A_{0,j}a_{j}k_{1}^{2}} \\ , \lambda_{3} &= \frac{6A_{1,j}k_{2}\chi_{0}(\beta_{j}+\gamma_{j})-4A_{0,j}A_{1,j}\chi_{0}}{3a_{j}k_{1}^{2}}, \ \lambda_{4} = -\frac{A_{1,j}^{2}\chi_{0}}{3a_{j}k_{1}^{2}}, \end{split}$$

$$\mu = \frac{4A_{0,j}^{4}\chi_{0}(b_{j}+c_{j}+d_{j})-4A_{0,j}^{3}k_{2}\chi_{0}(\beta_{j}+\gamma_{j})+A_{1,j}a_{j}k_{1}^{2}(A_{0,j}\lambda_{1}-A_{1,j}\lambda_{0})-4A_{0,j}^{2}a_{j}k_{2}^{2}\chi_{0}}{4A_{0,j}^{2}\chi_{0}}$$

 $\pm(\zeta - \zeta_0) = Q \int \frac{du}{\sqrt{\Gamma(u)}},$ (21) where  $Q = \sqrt{\frac{\chi_0}{\lambda_4}}, \quad \Gamma(u) = \sum_{i=0}^4 \frac{\lambda_i}{\lambda_4} u^i.$ 

Therefore the traveling wave solutions to Eq.(2) are

When 
$$\Gamma(u) = (u - \vartheta_1)^4$$
  
 $\psi(x, t) = \sqrt{A_{0,1} + A_{1,1}\vartheta_1 \pm \frac{A_{1,1}Q}{k_1x - 2a_1k_1k_2t - \zeta_0}} \times e^{i(-k_2x + \mu t + k_3)},$ 
(22)

$$\phi(x,t) = \sqrt{A_{0,2} + A_{1,2}\vartheta_1 \pm \frac{A_{1,2}Q}{k_1x - 2a_2k_1k_2t - \zeta_0}} \times e^{i(-k_2x + \mu t + k_3)}.$$
(23)  
When  $\Gamma(u) = (u - \theta)^3(u - \theta)$ , and  $\theta > \theta$ 

when 
$$\Gamma(u) = (u - v_1)^{-1} (u - v_2)$$
, and  $v_2 > v_1$   
 $\psi(x, t) = \sqrt{A_{0,1} + A_{1,1}v_1 + \frac{4A_{1,1}Q^2(v_2 - \vartheta_1)}{4Q^2 - [(\vartheta_1 - \vartheta_2)(k_1x - 2a_1k_1k_2t - \zeta_0)]^2}} \times e^{i(-k_2x + \mu t + k_3)}$ , (24)

 $\phi(x,t) = \sqrt{A_{0,2} + A_{1,2}\vartheta_1 + \frac{4A_{1,2}Q^2(\vartheta_2 - \vartheta_1)}{4Q^2 - [(\vartheta_1 - \vartheta_2)(k_1x - 2a_2k_1k_2t - \zeta_0)]^2}} \times e^{i(-k_2x + \mu t + k_3)}$ (25) When  $(u - \vartheta_1)^2 (u - \vartheta_2)^2$ 

$$\begin{split} \psi(x,t) &= \sqrt{A_{0,1} + A_{1,1}\vartheta_{L} + \frac{(-1)^{L+1}A_{1,1}(\vartheta_{1} - \vartheta_{2})}{e^{\frac{(\vartheta_{1} - \vartheta_{2})(k_{1}x - 2a_{1}k_{1}k_{2}t - \zeta_{0})}{Q}}} \times \\ e^{i(-k_{2}x + \mu t + k_{3})}, & (26) \\ \phi(x,t) &= \sqrt{A_{0,2} + A_{1,2}\vartheta_{L} + \frac{(-1)^{L+1}A_{1,2}(\vartheta_{1} - \vartheta_{2})}{e^{\frac{(\vartheta_{1} - \vartheta_{2})(k_{1}x - 2a_{1}k_{1}k_{2}t - \zeta_{0})}{Q}}} \times \\ e^{i(-k_{2}x + \mu t + k_{3})}, & (27) \\ \text{where } L = 1,2. & (27) \\ \text{where } L = 1,2. & (28) \\ \psi(x,t) &= \sqrt{A_{0,1} + A_{1,1}\vartheta_{1} - \frac{2A_{1,1}(\vartheta_{1} - \vartheta_{2})(\vartheta_{1} - \vartheta_{3})}{R1}} \times \\ e^{i(-k_{2}x + \mu t + k_{3})}, & (28) \\ \phi(x,t) &= \sqrt{A_{0,2} + A_{1,2}\vartheta_{1} - \frac{2A_{1,2}(\vartheta_{1} - \vartheta_{2})(\vartheta_{1} - \vartheta_{3})}{R1}} \times \\ e^{i(-k_{2}x + \mu t + k_{3})}, & (29) \\ \text{Where } R1 &= 2\vartheta_{1} - \vartheta_{2} - \vartheta_{3} + (\vartheta_{3} - \vartheta_{2}) \times \\ \cosh\left(\frac{(k_{1}\sqrt{(\vartheta_{1} - \vartheta_{2})(\vartheta_{1} - \vartheta_{3})})(k_{1}x - 2a_{1}k_{1}k_{2}t - \zeta_{0})}{Q}\right). \\ \text{Where } R1 &= 2\vartheta_{1} - \vartheta_{2} - \vartheta_{3} + (\vartheta_{3} - \vartheta_{2}) \times \\ \cosh\left(\frac{(k_{1}\sqrt{(\vartheta_{1} - \vartheta_{2})(\vartheta_{1} - \vartheta_{3})})(k_{1}x - 2a_{1}k_{1}k_{2}t - \zeta_{0})}{Q}\right). \\ \text{When } \Gamma &= (u - \vartheta_{1})(u - \vartheta_{2})(u - \vartheta_{3})(u - \vartheta_{4}), \text{ and } \\ \vartheta_{1} > \vartheta_{2} > \vartheta_{3} > \vartheta_{4} \\ \psi(x,t) &= \sqrt{A_{0,1} + A_{1,1}\vartheta_{2} + \frac{2A_{1,1}(\vartheta_{1} - \vartheta_{2})(\vartheta_{4} - \vartheta_{2})}{R2}} \times \\ e^{i(-k_{2}x + \mu t + k_{3})}, & (30) \\ \phi(x,t) &= \sqrt{A_{0,2} + A_{1,2}\vartheta_{2} + \frac{2A_{1,2}(\vartheta_{1} - \vartheta_{2})(\vartheta_{4} - \vartheta_{2})}{R2}} \times \\ e^{i(-k_{2}x + \mu t + k_{3})}, & (31) \\ \text{Where} \\ R2 &= \vartheta_{4} - \vartheta_{2} \\ \frac{(\vartheta_{1} - \vartheta_{4})sn^{2} \left(\pm \sqrt{(\vartheta_{1} - \vartheta_{3})(\vartheta_{2} - \vartheta_{4})(k_{1}x - 2a_{1}k_{1}k_{2}t - \zeta_{0}), m}\right)}{20} \end{pmatrix}$$

and  $m^2 = \frac{(\vartheta_2 - \vartheta_3)(\vartheta_1 - \vartheta_4)}{(\vartheta_1 - \vartheta_3)(\vartheta_2 - \vartheta_4)}$ . Note that  $\vartheta_i$ ,  $i = 1, \dots, 4$  are the roots of  $\Gamma(u) = 0$ . When  $A_{0,j} = -A_{1,j}\vartheta_1$  and  $\zeta_0 = 0$ , the solutions (22) - (31) are reduced to the following plane wave solutions:  $\psi(x,t) = \sqrt{\pm \frac{A_{1,1}Q}{k_1x - 2a_1k_1k_2t}} \times e^{i(-k_2x + \mu t + k_3)}$ , (32)  $\phi(x,t) = \sqrt{\pm \frac{A_{1,2}Q}{k_1x - 2a_1k_1k_2t}} \times e^{i(-k_2x + \mu t + k_3)}$ , (33)

$$\psi(x,t) = \sqrt{\frac{4A_{1,1}Q^{2}(\vartheta_{2}-\vartheta_{1})}{4Q^{2}-[(\vartheta_{1}-\vartheta_{2})(k_{1}x-2a_{1}k_{1}k_{2}t)]^{2}}} \times e^{i(-k_{2}x+\mu t+k_{3})}, \quad (34)$$
  
$$\phi(x,t) = \sqrt{\frac{4A_{1,2}Q^{2}(\vartheta_{2}-\vartheta_{1})}{4Q^{2}-[(\vartheta_{1}-\vartheta_{2})(k_{1}x-2a_{1}k_{1}k_{2}t)]^{2}}} \times e^{i(-k_{2}x+\mu t+k_{3})}, \quad (35)$$

© 2021 The Author(s). This open access article is distributed under a CC BY-NC 4.0 license. ISSN: online 2617-2186 print 2617-2178 singular soliton solutions:

$$\psi(x,t) = \sqrt{\frac{A_{1,1}(\vartheta_2 - \vartheta_1)}{2} (1 \mp \coth(X))} \times e^{i(-k_2 x + \mu t + k_3)}, \quad (36)$$

$$\phi(x,t) = \sqrt{\frac{A_{1,2}(b_2-b_1)}{2}} (1 \mp \coth(X)) \times e^{i(-k_2x+\mu t+k_3)}, \quad (37)$$
  
and bright soliton solutions:

$$\psi(x,t) = \left(\frac{D}{\sqrt{C + \cosh(B(k_1x - 2a_1k_1k_2t - \zeta_0))}}\right) \times e^{i(-k_2x + \mu t + k_3)}, \quad (38)$$

$$\begin{split} \phi(x,t) &= \left(\frac{D}{\sqrt{c + \cosh(B(k_1 x - 2a_1k_1k_2 t - \zeta_0))}}\right) \times e^{i(-k_2 x + \mu t + k_3)}, \quad (39)\\ \text{where } D &= \sqrt{\frac{2A_{1,j}(\vartheta_1 - \vartheta_2)(\vartheta_1 - \vartheta_3)}{(\vartheta_3 - \vartheta_2)}}, B = \frac{\sqrt{(\vartheta_1 - \vartheta_2)(\vartheta_1 - \vartheta_3)}}{Q},\\ C &= \frac{2\vartheta_1 - \vartheta_2 - \vartheta_3}{\vartheta_3 - \vartheta_2}, \ j = 1,2. \end{split}$$

The amplitude of the soliton is given by *D* where the inverse width of the soliton is given by *B*. The solitons will exist for  $A_{1,j} < 0$ . Furthermore, when  $A_{0,j} = -A_{1,j}$  and  $\zeta_0 = 0$ , Jacobi's elliptic function solutions (30), (31) are written as:

$$\psi(x,t) = \left(\frac{D_1}{\sqrt{C_1 + sn^2(B_L(k_1x - 2a_1k_1k_2t - \zeta_0))}}\right) \times e^{i(-k_2x + \mu t + k_3)},$$
(40)  
$$\phi(x,t) = \left(\frac{D_1}{\sqrt{C_1 + sn^2(B_L(k_1x - 2a_1k_1k_2t - \zeta_0))}}\right) \times e^{i(-k_2x + \mu t + k_3)},$$
(41)

where  $D_1 = \sqrt{\frac{A_{1,j}(\vartheta_1 - \vartheta_2)(\vartheta_4 - \vartheta_2)}{(\vartheta_1 - \vartheta_4)}},$  $B_L = \frac{(-1)^L \sqrt{(\vartheta_1 - \vartheta_3)(\vartheta_2 - \vartheta_4)}}{2Q}, \quad C_1 = \frac{2\vartheta_4 - \vartheta_2}{\vartheta_1 - \vartheta_4}, \text{ and } L = 1,2.$ 

**Remark-1:** When the modulus  $m \rightarrow 1$ , the singular optical soliton solutions are obtained as:

$$\begin{split} \psi(x,t) &= \left(\frac{D_1}{\sqrt{C_1 + \tanh^2(B_L(k_1x - 2a_1k_1k_2t - \zeta_0))}}\right) \times \\ e^{i(-k_2x + \mu t + k_3)}, & (42) \\ \phi(x,t) &= \left(\frac{D_1}{\sqrt{C_1 + \tanh^2(B_L(k_1x - 2a_1k_1k_2t - \zeta_0))}}\right) \times \\ e^{i(-k_2x + \mu t + k_3)}, & (43) \end{split}$$

where  $\vartheta_3 = \vartheta_4$ .

**Remark-2:** When the modulus  $m \rightarrow 0$ , singular-periodic solutions are obtained as:

$$\psi(x,t) = \left(\frac{D_1}{\sqrt{c_1 + \sin^2(B_L(k_1x - 2a_1k_1k_2t - \zeta_0))}}\right) \times e^{i(-k_2x + \mu t + k_3)},$$
(44)

$$\phi(x,t) = \left(\frac{D_1}{\sqrt{C_1 + \sin^2(B_L(k_1x - 2a_1k_1k_2t - \zeta_0))}}\right) \times e^{i(-k_2x + \mu t + k_3)},$$
(45)  
where  $\vartheta_2 = \vartheta_2$ .

#### CONCLUSION

The coupled system corresponding to the Gerdjikov-Ivanov equation, without FWM in birefringent fibers, was considered on account of acquiring optical soliton solutions. Bright soliton, and singular soliton solutions, were presented by the extended trial function scheme. Additional solutions, which are singular and singular-periodic soliton solutions, were obtained using the limiting of the modulus of ellipticity of the Jacobi elliptic function. Subsequently, by virtue of this paper, four-wave mixing terms (FWM) will be added to the model discussed in this article, and results will be reported accordingly.

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# الحلول البصرية اللامتغيرة زمنيا لمعادلة جيردجيكوف ايفانوف بدون تداخل رباعي الموجات في الألياف ثنائية الانكسار باستخدام طريقة الدالة التجريبية الممتدة

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**المستخلص**: بدون تداخل رباعي الموجات، طريقة الدالة التجريبية الممتدة استخدمت للحصول على حلول بصرية لا متغيرة زمنيا للنظام المزدوج المقابل لمعادلة جيردجيكوف ايفانوف. الإجراء يكشف حلول بصرية مفردة، حلول بصرية ساطعة، وحلول في غاية الأهمية في صيغة دالة جاكوبي الإهليجية، وفي نهايات الدالة الإهليجية نحصل على حلول بصرية مفردة، وحلول بصرية مفردة دورية جنبا إلى جنب مع معايير وجودها.

الكلمات المفتاحية: ألياف ثنائية الانكسار، نموذج جيردجيكوف ايفانوف المزدوج بدون تداخل رباعي الموجات، طريقة الدالة التجريبية الممتدة، حلول بصرية.

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