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Normal Mode Analysis of Generalized Magneto-Thermoelastic Medium with Initial Stress Under Green-Naghdi Theory

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Abstract: The normal mode analysis method was used to study the effect of both the initial stress and the magnetic field on a thermally elastic body. This method is used to obtain the exact expressions for the considered variables. Some particular cases are also discussed in the context of the problem. The generalized thermal elasticity equations were reviewed under the influence of the basic initial stress and the magnetic field using the theory (Green-Naghdi) of the second and third types (the second type with no energy dispersion and the third type with energy dispersion). The different physical quantities were illustrated in the presence and absence of both the initial stress and the magnetic field. The results of this research show the extent of difference between the second and third types of Green and Naghdi's theory. All results and figures were obtained using (MATLAB R2013a) program.

Keywords: Generalized Thermo-Elasticity; Magnetic Field; Initial Stress; Normal Mode Analysis; Green and Naghdi Theory.

INTRODUCTION

The generalized theory of thermoelasticity is one of the modified versions of the classical uncoupled and coupled theory of thermoelasticity and has been developed in order to remove the paradox of physical impossible phenomena of the infinite velocity of thermal signals in the classical coupled thermoelasticity. (Hetnarski & Ignaczak, 1999) examined five generalizations of the coupled theory of thermoelasticity. The first generalization was proposed by (Lord & Shulman, 1967), which involves one relaxation time for a thermoelastic process. The second generalization is due to (Green & Lindsay, 1972) which takes into account two relaxation times. The third generalization of the coupled theory of thermoelasticity was introduced by (Green & Naghdi, 1993), who developed different theories labeled type I, type II, and type III. The (G-N I) theory in the linearized theory is equivalent to the classical coupled thermoelasticity theory. The (G-N II) theory does not admit energy dissipation, while the third (G-N III) theory admits dissipation of energy. The heat flux is a combination of type I and type II. Both type II and type III theories imply a finite speed of propagation for heat waves. (Bargmann & Steinmann, 2006) investigated the (G-N) approach for modeling the phenomenon of second sound.(Othman & Atwa, 2011; Othman & Atwa, 2012; Othman et al., 2013b; Othman & Kumar, 2009), has discussed different problems for various materials with different effects using the (G-N) theory. The fourth generalization of the coupled theory of thermoelasticity was developed by (Chandrasekharaiah, 1998; Tzou, 1995).

Initial stress in solids has a significant influence on the mechanical response of the material from an initially stressed configuration and has applications in geophysics, engineering structures, and the behavior of soft biological tissues. Initial stress arises from pro-

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cesses, such as manufacturing or growth, and is present in the absence of applied loads. (Montanaro, 1999) formulated the isotropic thermoelasticity with hydrostatic initial stress. (Ailawalia et al., 2009; Othman & Song, 2007; Singh, 2008; Singh et al., 2006), and many others have applied (Montanaro, 1999) theory to study the plane harmonic waves in the context of generalized thermoelasticity. (Othman & Atwa, 2012) investigate the effect of initial stress under the Green-Naghdi (G-N) theory for different cases in thermoelasticity. (Ailawalia & Narah, 2009) studied the effect of hydrostatic initial stress and rotation in a generalized thermoelastic medium. (Othman & Edeeb, 2016) studied the Effect of Initial Stress on Generalized Magneto-thermoelasticity Medium with Voids: A Comparison of Different Theories. (Abd-Elaziz et al., 2019) studied the On the Effect of Thomson and Initial Stress in a Thermo-Porous Elastic Solid under G-N Electromagnetic Theory.

The theory of magneto-thermoelasticity is concerned with the interacting effects of the applied magnetic field on the elastic and thermoelastic deformations of a solid body. This theory has aroused much interest in many industrial appliances, particularly in nuclear devices where there exists a primary magnetic field; various investigations are to be carried out by considering the interaction between magnetic, thermal, and strain fields. Analyses of such problems also influence various applications in biomedical engineering as well as in different geomagnetic studies. The development of the interaction of electromagnetic field, the thermal field, and the elastic field is available in many works such as (Abd-Alla et al., 2003; Choudhuri & Debnath, 1985; Othman & Song, 2006; Paria, 1966; Sherief & Helmy, 2002) studied the effect of rotation on the reflection of magneto-thermoelastic waves under thermoelasticity without energy dissipation with the (G-N) theory of type II. (Othman & Kumar, 2009) studied the reflection of magneto-thermoelastic waves with temperature-dependent properties in the context of generalized thermoelasticity with (G-N) theory of type II, i.e. without energy dissipation, and other models of thermoelasticity. (Othman & Atwa, 2011) studied the effect of the magnetic field on the two-dimensional problem of generalized thermoelasticity without energy dissipation. (Othman et al., 2013a) studied the generalized magneto-thermo-microstretch elastic solid under a gravitational field with energy dissipation. Recently (Othman et al., 2013b) studied the effect of magnetic field and rotation on generalized thermo-microstretch elastic solid for a mode-I crack using (G-N) theory. (Atwa, 2014) studied the generalized magneto-thermoelasticity with two temperatures and initial stress under Green–Naghdi theory. (Abo-Dahab et al., 2017) studied A Two-Dimensional Problem with Rotation and Magnetic Field in the Context of Four Thermoelastic Theories, the normal-mode analysis method was applied to obtain the exact solutions for the physical problem.

FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

Consider an isotropic, homogeneous, linear, thermally, and electrically conducting thermoelastic half-space $(x \geq 0, \ -\infty \leq y \leq \infty)$. The rectangular Cartesian coordinate system (x, y, z) , having originated on the surface $z = 0$, for the two dimensional problem assume the dynamic displacement vector as $u = (u, v, 0)$. The surface $(x = 0)$ of the half-space is taken to be traction-free and subjected to mechanical and thermal loads. All the considered functions are assumed to be bounded as $x \to \infty$.. The whole body is at a constant temperature T_0.. Consider also that the orientation of the primary magnetic field $\boldsymbol{H} = (0, 0, H_0)$ is towards the positive direction of $z-$axis. Due to the application of this magnetic field, an induced magnetic field $\boldsymbol{h}$ and an induced electric field $\boldsymbol{E}$. arise in the medium .All the considered functions will

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depend on time t and the coordinates x and y. So the displacement vector $\boldsymbol{u}$ has the components

$$u_x = u(x,z,t), \quad u_y = v(x,z,t), \quad u_z = 0. \quad (1)$$

The variation of the magnetic and electric fields are a perfectly conducting slowly moving medium and are given by Maxwell's equations:

$$\operatorname{curl} \boldsymbol{h} = \boldsymbol{J} + \varepsilon_0 \dot{\boldsymbol{E}}, \quad (2)$$

$$\operatorname{curl} \boldsymbol{E} = -\mu_0 \dot{\boldsymbol{h}}, \quad (3)$$

$$\boldsymbol{E} = -\mu_0 (\dot{\boldsymbol{u}} \;\; \boldsymbol{H} \quad (4)$$

$$\operatorname{div} \boldsymbol{h} = 0. \quad (5)$$

From the above equations, one can obtain

$$\boldsymbol{E} = \mu_0 H_0 (\dot{v}, 0, -\dot{u} \quad (6)$$

$$\boldsymbol{h} = (0, 0, -H_0\, e), \quad (7)$$

$$\boldsymbol{J} = (-h_{,y} - \varepsilon_0 \mu_0 H_0 \ddot{v}, 0, h_{,x} + \varepsilon_0 \mu_0 H_0 \ddot{u} \quad (8)$$

The constitutive relations are given by

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}[\lambda e_{kk} - \beta \frac{\partial T}{\partial x}] - P(\delta_{ij} + \omega_{ij}) \quad (9)$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad \omega_{ij} = \frac{1}{2}(u_{j,i} - u_{i,j}), \quad (10)$$

The equation of motion has the form

$$\sigma_{ji,j} + F_i = \rho \ddot{u} \qquad i,j = 1,2,3. \quad (11)$$

Where F_i is the Lorentz force and is given by: $F_i = \mu_0 (\boldsymbol{J} \times \boldsymbol{H})_i$. (12)

From equations (8) and (11), Lorentz force is obtained

$$\boldsymbol{F} = (F_x, F_y, F_z) = (\mu_0 H_0^2 e_{,x} - \varepsilon_0 \mu_0^2 H_0^2 \ddot{u} \quad {}_0^2 e_{,y} - \varepsilon_0 \mu_0^2 H_0^2 \ddot{v}, 0). \quad (13)$$

Substituting from equations (9) and (13) into equation (11), the equations of motion can be written as follows

$$(\mu - \frac{P}{2})\nabla^2 u + (\lambda + \mu + \mu_0 H_0^2 + \frac{P}{2})e_{,x} - \beta T_{,x} = \rho(1 + \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho})\ddot{u} \quad (14)$$

$$(\mu - \frac{P}{2})\nabla^2 v + (\lambda + \mu + \mu_0 H_0^2 + \frac{P}{2})e_{,y} - \beta T_{,y} = \rho(1 + \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho})\ddot{v}. \quad (15)$$

The equation of heat conduction has the form

$$K\nabla^2 T + k^* \nabla^2 \dot{T} \quad \rho C_E \ddot{T} \quad \rho T_0 \ddot{e} \quad (16)$$

Where, σ_{ij} are the stress tensor components, e_{ij} are the strain tensor components, ω_{ij} is the rotation tensor, $e = e_{kk}$ is the cubic dilatation, δ_{ij} is Kronecker's delta, u_i is the displacement vector, λ, μ are the elastic constants, T is the absolute temperature, T_0 is the temperature of medium in its natural state assumed to be such that $|(T - T_0)/T_0| < 1$, ε_0, μ_0 are the electric and magnetic permeability respectively, $\boldsymbol{J}$ is the current density vector, $\boldsymbol{E}$ is the induced electric field vector, $\boldsymbol{h}$ is the induced magnetic field vector, H_0 is a constant magnetic field, P is the initial stress, λ, μ are Lame's constants, $\beta = (3\lambda + 2\mu)\alpha_t$, α_t is the coefficient of linear thermal expansion, ρ is the density, C_E is the specific heat at constant strain, k is the coefficient of thermal conductivity, k^* is the material constant characteristic of the theory, and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

When $k^* \to 0$ then equation (16) reduces to the heat conduction equation in (G-N) theory (of type II),

$$\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j. \quad \text{and} \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

The components of stress tensor are

$$\sigma_{xx} = \lambda(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) + 2\mu \frac{\partial u}{\partial x} - \beta T - p, \quad (17)$$

$$\sigma_{yy} = \lambda(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) + 2\mu \frac{\partial v}{\partial y} - \beta T - p, \quad (18)$$

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$$\sigma_{zz} = \lambda(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) - \beta T - p, \quad (19)$$

$$\sigma_{xy} = \mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) - \frac{p}{2}(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}). \quad (20)$$

The basic governing equations of linear magnetic thermoelastic materials under influence of the initial stress become

$$(\mu - \frac{p}{2})\nabla^2 u + (\lambda + \mu + \frac{p}{2})\frac{\partial e}{\partial x} - \beta\frac{\partial T}{\partial x} + \mu_0 H_0^2 \frac{\partial e}{\partial x} - \mu_0^2 H_0^2 \varepsilon_0 \frac{\partial^2 u}{\partial t^2} = \rho\frac{\partial^2 u}{\partial t^2}, \quad (21)$$

$$(\mu - \frac{p}{2})\nabla^2 v + (\lambda + \mu + \frac{p}{2})\frac{\partial e}{\partial y} - \beta\frac{\partial T}{\partial y} + \mu_0 H_0^2 \frac{\partial e}{\partial y} - \mu_0^2 H_0^2 \varepsilon_0 \frac{\partial^2 v}{\partial t^2} = \rho\frac{\partial^2 v}{\partial t^2}, \quad (22)$$

$$K\nabla^2 T + k^*\frac{\partial}{\partial t}\nabla^2 T = \rho C_E \frac{\partial^2 T}{\partial t^2} + \beta T_0 \frac{\partial^2 e}{\partial t^2}. \quad (23)$$

Where, $h = -H_0 e$

For the purpose of numerical evaluation, dimension variables are introduced.

$$(u', v') = \frac{\omega_1^*}{c_1}(u, v), \; T' = \frac{T}{T_0}, \quad (x', y') = \frac{\omega_1^*}{c_0}(x, y), \; \sigma'_{xx} = \frac{\sigma_{xx}}{\mu},$$
$$\sigma'_{xy} = \frac{\sigma_{xy}}{\mu}, \; ,\sigma'_{ij} = \frac{\sigma_{ij}}{\mu_0}\sigma_{ij}, \quad (24)$$
$$t' = \omega_1^* t, \; c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad \omega_1^* = \frac{\rho C_E c_1^2}{k}, \; h' = \frac{h}{H_0},$$
$$\beta = (3\lambda + 2\mu)\alpha_t, \quad p' = \frac{p}{\mu}.$$

Equations (21) - (23), with the help of non-dimensional variables (24) may be recast into the dimensionless form after dropping primes for convenience as:

$$\nabla^2 u + E_1\frac{\partial e}{\partial x} - E_2\frac{\partial T}{\partial x} = E_3\frac{\partial^2 u}{\partial t^2}, \quad (25)$$

$$\nabla^2 v + E_1\frac{\partial e}{\partial y} E_2\frac{\partial T}{\partial y} = E_3\frac{\partial^2 v}{\partial t^2}, \quad (26)$$

$$\varepsilon_1\nabla^2 T + \varepsilon_2\frac{\partial}{\partial t}\nabla^2 T = \frac{\partial^2 T}{\partial t^2} + \varepsilon_3\frac{\partial^2 e}{\partial t^2}. \quad (27)$$

Here,

$$E_1 = \frac{2\lambda + \mu(2+p) + 2\mu_0 H_0^2}{\mu(2-p)},$$

$$E_2 = \frac{2\beta T_0}{\mu(2-p)}, \quad E_3 = \frac{2\mu_0 H_0^2 \varepsilon_0 c_1^2 + 2\rho c_1^2}{\mu(2-p)},$$

$$\varepsilon_1 = \frac{K}{c_1^2 \rho c e}, \quad \varepsilon_2 = \frac{K\omega_1^*}{c_1^2 \rho c e}, \quad \varepsilon_3 = \frac{\beta}{\rho c e}.$$

Where $\varepsilon_1, \varepsilon_2$, and ε_3 are the coupling constants. Using the expression relating displacement components $u(x, y, t)$ and $v(x, y, t)$ to the scalar potential functions $\psi_1(x, y, t)$ and $\psi_2(x, y, t)$ in dimensionless form.

$$u = \frac{\partial\psi_1}{\partial x} + \frac{\partial\psi_2}{\partial y}, \text{ and } v = \frac{\partial\psi_1}{\partial y} - \frac{\partial\psi_2}{\partial x}, \quad (28)$$

$$e = \nabla^2\psi_1, \text{ and } (\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}) = \nabla^2\psi_2. \quad (29)$$

By substituting from Eq. (29) in Eqs. (25)-(27), this yields

$$(1 + E_1)\nabla^2\psi_1 - E_2 T = E_3\frac{\partial^2}{\partial t^2}\psi_1, \quad (30)$$

$$[\nabla^2 - E_3\frac{\partial^2}{\partial t^2}]\psi_2 = 0, \quad (31)$$

$$\varepsilon_1\nabla^2 T + \varepsilon_2\frac{\partial}{\partial t}\nabla^2 T = \frac{\partial^2 T}{\partial t^2} + \varepsilon_3\frac{\partial^2 e}{\partial t^2}\nabla^2\psi_1. \quad (32)$$

Then the components of stress tensor will be

$$\sigma_{xx} = E_4(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) + 2\frac{\partial u}{\partial x} - E_2 T - p, \quad (33)$$

$$\sigma_{yy} = E_4(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) + 2\frac{\partial v}{\partial y} - E_2 T - p, \quad (34)$$

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$$\sigma_{zz} = E_4(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) - E_2 T - p, \quad (35)$$

$$\sigma_{xy} = (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) - \frac{p}{2}(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}). \quad (36)$$

Where $E4 = \frac{\lambda}{\mu}$,

NORMAL MODE ANALYSIS

The applied methodology to the system was the normal mode analysis to obtain the exact expressions for the used physical variables. The solution of the considered physical variables can be decomposed in terms of the normal mode as the following form

$$[u, v, T, \psi_1, \psi_2,, \sigma_{ij}](x, y, t) = [u^*, v^*, T^*, \psi_1^*, \psi_2^*, \sigma_{ij}^*](y)\exp[i(\omega t + ax)], \quad (37)$$

Where $[u^*, v^*, T^*, \psi_1^*, \psi_2^*, \sigma_{ij}^*](y)$ are the amplitudes of the function, ω is the complex time constant, $i = \sqrt{-1}$ and a is the wave number in x-direction.

Using Eq. (37) into Eqs. (30)-(32), yields the following,

$$(D^2 - F_2)\psi_1^* - F_3 T^* = 0, \quad (38)$$

$$(D^2 - F_4)\psi_2^* = 0, \quad (39)$$

$$F_5(D^2 - a^2)\psi_1^* + (D^2 - F_6)T^* = 0. \quad (40)$$

Where,

$$D = \frac{d}{dy}, \quad F_1 = 1 + E_1, \quad F_2 = \frac{F_1 a^2 - E_3\omega^2}{F_1},$$

$$F_3 = \frac{E_2}{F_1}, \; F_4 = a^2 - E_3\omega^2, \; F_5 = \frac{\varepsilon_3\omega^2}{\varepsilon_1 + \varepsilon_2\omega},$$

Eliminating $\psi_1^*(y)$ and $T^*(y)$ between Eqs. (38) - (40), yields the following fourth order ordinary differential equations for $\psi_1^*(y)$ and $T^*(y)$:

$$[D^4 - AD^2 + B]\{\psi_1^*(y), T^*(y)\} = 0. \quad (41)$$

Equation (41) can be factored as

$$(D^2 - k_1^2)(D^2 - k_2^2)\{\psi_1^*(y), T^*(y)\} = 0. \quad (42)$$

Where $k_n^2 (n = 1, 2)$ are the roots of the characteristic equation of Eq. (41),

$$A = F_2 + F_6 + -F_3 F_5,$$

$$B = F_2 F_6 - F_3 F_5 a^2, \quad m^2 = F_4 = a^2 - E_3\omega^2.$$

The solution of Eqs. (41) and (39) have the form

$$\psi_1^*(y) = \sum_{n=1}^{2} G_n e^{-k_n y}, \quad (43)$$

$$T^*(y) = \sum_{n=1}^{2} L_{1n} G_n e^{-k_n y}, \quad (44)$$

$$\psi_2^* = G_3 e^{-my}. \quad (45)$$

Where $G_n (n = 1, 2, 3)$ are some parameters and $L_{1n} = \frac{k_n^2 - F_2}{F_3}$.

Substituting from Eqs. (43)-(45) and (37) in Eqs. (28), (33)-(35) respectively, the displacement and stress components take the form

$$u^* = (\sum_{n=1}^{2} i\, a G_n e^{-k_n y} - m\, G_3 e^{-my})e^{(\omega t + iax)}, \quad (46)$$

$$v^* = (\sum_{n=1}^{2} -k_n G_n e^{-k_n y} - i\, a G_3 e^{-my})e^{(\omega t + iax)}, \quad (47)$$

$$\sigma_{xx}^* = (\sum_{n=1}^{2} L_{2n} G_n e^{-k_n y} - b_1 G_3 e^{-my})e^{i(\omega t + ax)} - p, \quad (48)$$

$$\sigma_{yy}^* = (\sum_{n=1}^{2} L_{3n} G_n e^{-k_n y} + b_1 G_3 e^{-my})e^{(\omega t + iax)} - p, \quad (49)$$

$$\sigma_{xy}^* = (\sum_{n=1}^{2} L_{4n} G_n e^{-k_n y} + b_2 G_3 e^{-my})e^{(\omega t + iax)}, \quad (50)$$

Where,

$$L_{2n} = (k_n^2 - a^2)E_4 + 2k_n^2 - E_2 L_{1n},$$

$$L_{3n} = L_{2n} = (k_n^2 - a^2)E_4 - 2a^2 - E_2 L_{1n},$$

$$L_{4n} = 2iak_n, \quad b_1 = 2iam,$$

$$b_2 = (a^2 + m^2) - \frac{p}{2}(m^2 - a^2).$$

THE BOUNDARY CONDITIONS

In this section, the boundary conditions at $y = 0$, needs to be considered, in order to determine the constants G_n $(n = 1, 2, 3)$:

(1) The mechanical boundary conditions

$$\sigma_{yy} = -P_1 e^{(\omega t + iax)}, \qquad \sigma_{xy} = 0, \quad (51)$$

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(2) The thermal boundary condition that the surface of the half-space is subjected to

$$T = P_2 e^{(\omega t + i a x)}. \tag{52}$$

Where P_1 is the magnitude of the applied force on the half-space and P_2 is the applied constant temperature to the boundary.

Using the expressions of the variables into the above boundary conditions (51), (52) produces,

$$\sum_{n=1}^{2} L_{3n} G_n + b_1 G_3 = -p_1, \tag{53}$$

$$\sum_{n=1}^{2} L_{4n} G_n + b_1 G_3 = 0, \tag{54}$$

$$\sum_{n=1}^{3} L_{1n} G_n = P_2. \tag{55}$$

Invoking boundary conditions (53)-(55) at the surface $y = 0$ of the plate, yields a system of three equations. After applying the inverse of the matrix method, one can get the values of the three constants G_n $(n = 1, 2, 3)$.

$$\begin{pmatrix} G_1 \\ G_2 \\ G_3 \end{pmatrix} = \begin{pmatrix} L_{31} & L_{32} & b_1 \\ L_{41} & L_{42} & b_2 \\ L_{11} & L_{12} & 0 \end{pmatrix}^{-1} = \begin{pmatrix} -P_1 \\ 0 \\ P_2 \end{pmatrix}. \tag{56}$$

Hence, obtaining the expressions for the displacements, the temperature distribution, and the other physical quantities of the plate surface.

$\lambda = 2.17 \times 10^{10}\ N/m^2$,
$\mu = 3.278 \times 10^{10}\ N/m^2$
$K = 1.7 \times 10^2\ W/m\,\text{deg}$,
$\alpha_t = 1.78 \times 10^{-5} N/m^2$,
$\rho = 1.74 \times 10^3\ kg/m^3$,
$C_E = 1.04 \times 10^3\ J/kg\,\text{deg}$, $T_0 = 298\,K$,
$\beta = 2.68 \times 10^6 N/m^2\,\text{deg}$, $\omega_1^* = 3.58 \times 10^{11}/s$.

The Magnetic field parameters were
$H_0 = 10^8$, $\mu_0 = 4\pi \times 10^{-7}\ H/M$,
$\varepsilon_0 = 8.85418717 \times 10^{-12} F/M$.

The comparisons were carried out for
$x = 0.5$, $t = 0.3$, $\omega = \zeta_0 + i\,\zeta_1$, $\zeta_0 = -0.7$, $\xi_1 = 0.1$, $p_1 = -0.1$, $p_2 = 0.2$, $a = 0.5$, $0 \le y \le 25$.

The comparisons have established for two cases

(i) With and without magnetic field

$[(H_0 = 10^8, 0),\quad p = 1,\quad t = 0.3]$.

(ii) With and without initial stress

$[(p = 1, 0),\quad H_0 = 10^8,\quad t = 0.3]$.

The above numerical technique, was used for the distribution of the real parts of the displacement components u and v, the temperature distribution T, the stress components σ_{xx}, σ_{yy} and σ_{xy} with the distance for (G-N) theory of both types II and III with and without the magnetic field ($H_0 = 0, 10^8$) during $p = 1$, and $t = 0.3$ in figures (1-6).

Figures (7-12) clarify the distribution of the real parts of the displacement components u and v, the temperature T, the stress components with the distance y for (G-N) theory of both types II and III with and without the initial stress ($p = 1, 0$) during $H_0 = 10^8$, and $t = 0.3$. Figures (1-12) are graphically represented changes in the behavior of the physical quantities against distance y in 2D.

Fig. 1 depicts that the distribution of the vertical displacement u, in the context of both types II and III, always begins from positive values for $H_0 = 10^8, 0$ and begins from negative values for $H_0 = 10^8$ of type III. It was observed that the displacement u increases with the increase of the magnetic field for $y > 0$. The distributions of u is directly proportional to the magnetic field.

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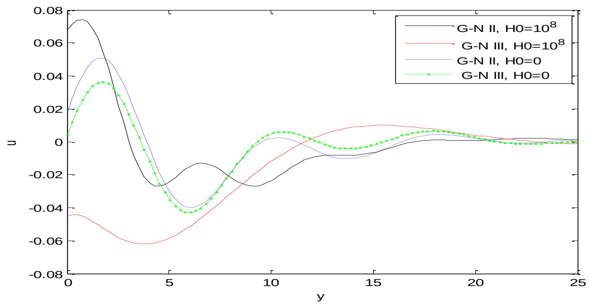

Fig.(1) Horizontal displacement distribution u in the absence and presence of the magnetic field.

Fig. 2 depicts the displacement distribution v, in the context of both types II and III for $H_0 = 10^8, 0$ it was observed that the distributions of v decrease with the increase of the magnetic field for $y > 0$. The distributions of v are inversely proportional to the magnetic field.

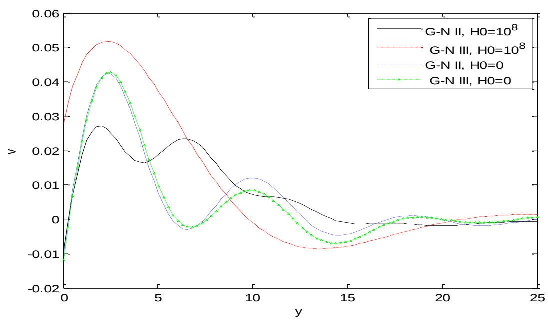

Fig.(2) Vertical displacement distribution v in the absence and presence of the magnetic field.

Fig. 3 explains that the distribution of temperature T begins from a positive value (which is the same point) in case of $H_0 = 0, 10^8$, in the context of both types II and III of (G-N), and takes the form of a wave until it develops to zero.

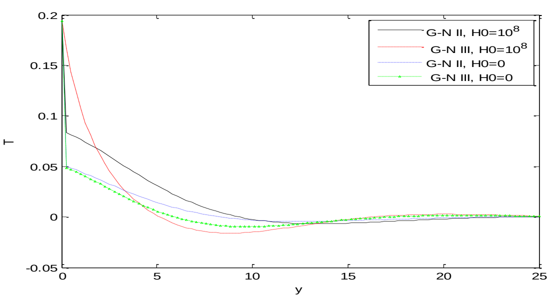

Fig. (3) Thermodynamic temperature distribution T in the absence and presence of magnetic field.

Fig. 4 determines the distribution of the stress component σ_{xx} in the case of $H_0 = 10^8$, and $H_0 = 0$, in the context of both types II and III. It was noted that the distribution of σ_{xx} decreases with the increase of the magnetic field value for $y > 0$.

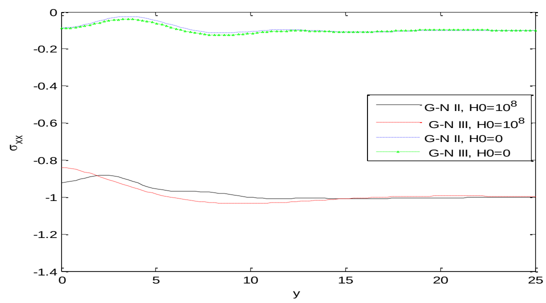

Fig. (4) Distribution of stress component σ_{xx} in the absence and presence of the magnetic field.

Fig. 5 shows the distribution of the stress component σ_{yy} in the case of $H_0 = 10^8, 0$, in the context of both types II and III. It was observed that the distribution of σ_{yy} decreases with the increase of the magnetic field value for $y > 0$.

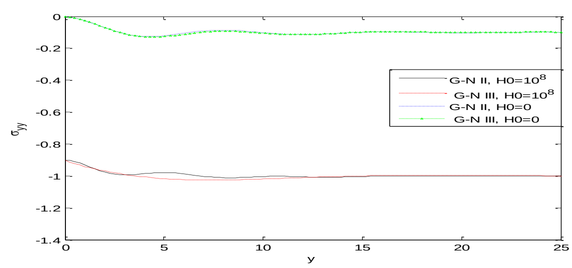

Fig. (5) Distribution of stress component σ_{xy} in the absence and presence of the magnetic field.

Fig. 6 explains the distribution of stress component σ_{xy} which begins from zero in the case of $H_0 = 10^8$, $H_0 = 0$, in the context of both types II and III. It was observed that the magnetic field has an effect on σ_{xy}, while the distribution of σ_{xy} increases with the increase of the magnetic field value for $y > 0$.

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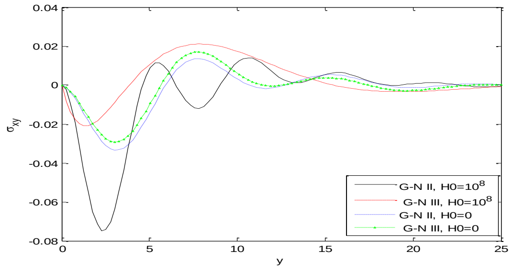

Fig. (6) Distribution of stress component σ_{xy} in the absence and presence of magnetic field.

Figs. 7 and 8 show the distribution of displacement components u and v in the case of $p=1$ and $p=0$, in the context of both types II and III. It was noted that the distributions of u and v respectively increase with the increase of the initial stress for $y > 0$. The distributions of u and v are directly proportional to the initial stress.

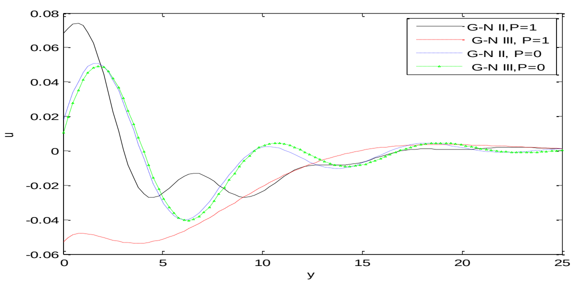

Fig. (7) Distribution of the displacement component u with and without initial stress.

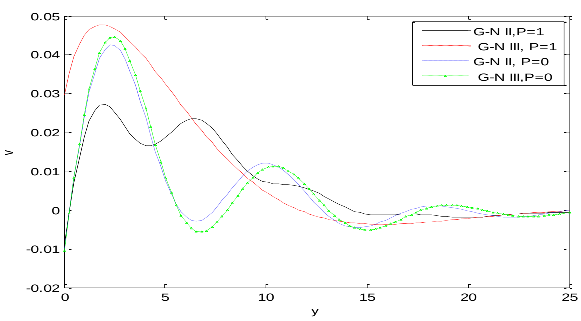

Fig. (8) Distribution of the displacement component v with and without initial stress.

Figs. 9 explains the distribution of temperature T which begins from zero in the case of $p=1$ and $p=0$, in the context of both types II and III. It was noticed that T decreases with the increase of the initial stress for $y > 0$.

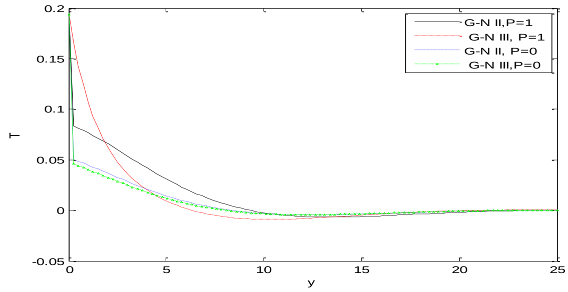

Fig. (9) Temperature distribution T with and without initial stress.

Figs. 10 and 11 depict the behavior of σ_{xx} and σ_{yy} in the context of both types II and III which always begin from positive values for $p=1, 0$, and begin from negative for $p=1$ in type II. It was observed that stress components σ_{xx} and σ_{yy} increase with the increase of the initial stress for $y > 0$.

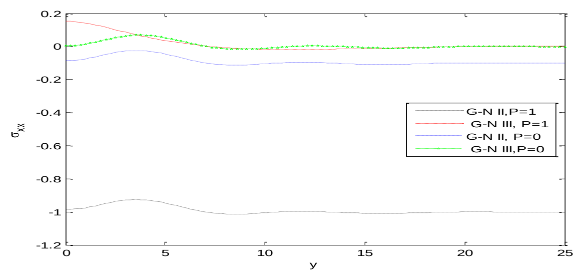

Fig. (10) Distribution of the stress component σ_{xx} with and without initial stress.

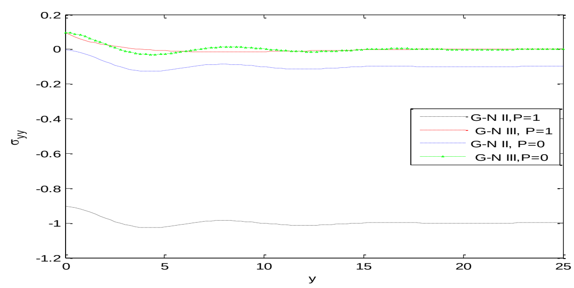

Fig. (11) Distribution of the stress component σ_{yy} with and without initial stress.

Figs. 12 demonstrates that the distribution of the stress component σ_{xy}, in the context of both types II and III begins from zero and satisfies the boundary conditions at $p=1$, and $p=0$. In the context of both types II and III. It was noted that the stress component of σ_{xy},

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increases with the increase of the initial stress values for $y > 0$.

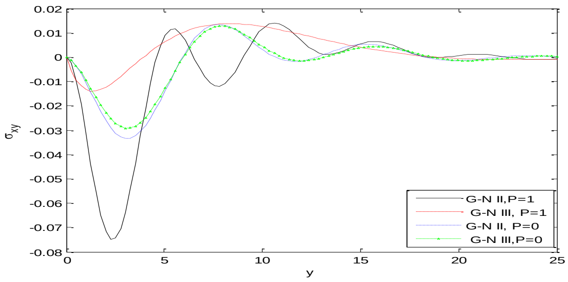

Fig. (12) Distribution of the stress component σ_{xy} with and without initial stress.

3D curves are representing $y = 0$ the complete relation between the physical variables and both of the components of the distance as shown in Figures (13-18) in the presence of the magnetic field $H_0 = 10^8$ and the initial stress proprieties $p = 1$, at $t = 0.3$ in the context of (G-N) theory of type III. These figures are very important in studying the dependence of these physical quantities on the vertical component of the distance. The obtained curves are highly dependent on the vertical distance from the origin, and all the physical quantities are moving in the wave propagation.

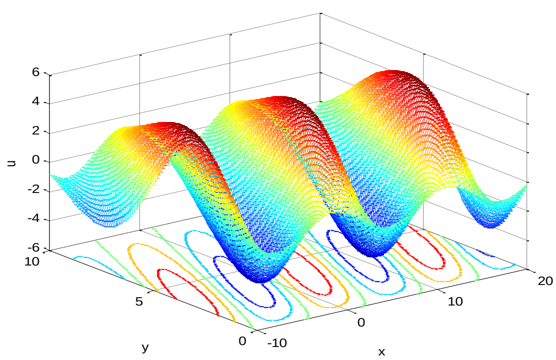

Fig. 13 (3D) Horizontal component of displacement u against both components of distance based on G-N type III at $H_0 = 10^8$ and $P = 1$.

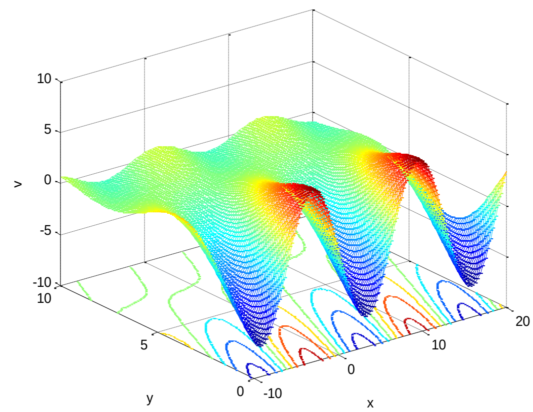

Fig. 14 (3D) Vertical component of displacement v against both components of distance based on G-N type III at $H_0 = 10^8$ and $P = 1$.

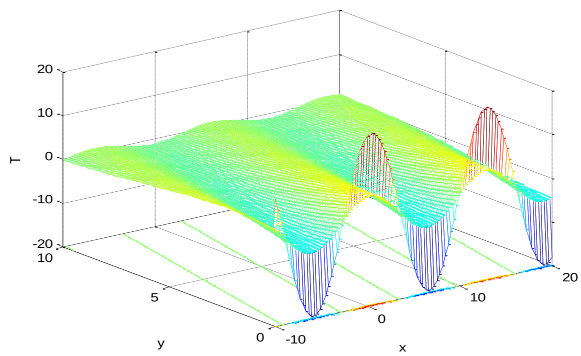

Fig. 15 (3D) Thermodynamic temperature distribution T against both components of distance based on G-N type III at $H_0 = 10^8$ and $P = 1$.

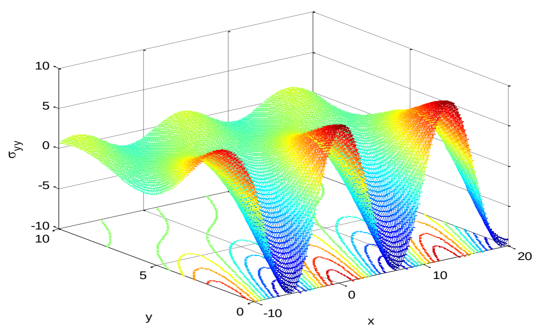

Fig. 16 (3D) Distribution of the stress component σ_{yy} against both components of distance based on G-N type III at $H_0 = 10^8$ and $P = 1$.

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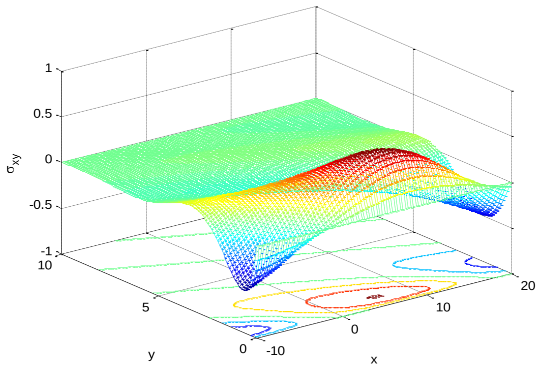

Fig. 17 (3D) Distribution of the stress component σ_{xy} against both components of distance based on G-N type III at $H_0 = 10^8$ and $P = 1$.

CONCLUDING REMARKS

By comparing the figures obtained under the (G-N) theory in the context of both types II and III, important phenomena are observed: the values of all physical quantities converge to zero with increasing distance *y,* all functions are continuous, and all physical quantities satisfy the boundary conditions. Also, analysis of the components of displacement, stresses, the temperature distribution due to the initial stress, and the magnetic field for thermoelastic solid with magnetic field under the initial stress is an interesting problem of mechanics. Normal mode analysis technique has been used, which applies to a wide range of problems in thermoelasticity. The value of all physical quantities converges to zero, with an increase in distance and all functions are continuous y. It was observed that the magnetic field and initial stress have a significant role in all considered physical quantities, as the amplitudes of these quantities vary (increasing or decreasing) with the increase of the initial stress and magnetic field.

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تحليل الوضع الطبيعي للوسط المغناطيسي الحراري المرن المعمم مع إجهاد أولي في ظل نظرية (جرين وناخدي)

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المستخلص: تم استخدام طريقة تحليل الوضع الطبيعي في دراسة تأثير كل من الإجهاد الأساسي الهيدروستاتيكي، والمجال المغناطيسي على جسم مرن حرارياً، وهذه الطريقة تستخدم للحصول على التعبيرات الدقيقة للمتغيرات المدروسة، وتناقش بعض الحالات الخاصة أيضا في سياق المشكلة . حيث تم استعراض معادلات المرونة الحرارية المعممة تحت تأثير الإجهاد الأساسي الهيدروستاتيكي، والمجال المغناطيسي باستخدام نظرية (جرين وناخدي) من النوعين الثاني والثالث (النوع الثاني مع عدم تشتت للطاقة والنوع الثالث مع تشتت للطاقة)، وتم رسم الكميات الفيزيائية المختلفة في حالة وجود، وعدم وجود كل من الضغط الهيدروستاتيكى، والمجال المغناطيسي. والنتائج التي خرج بها هذا البحث توضح مدى الفرق بين النوعين الثاني والثالث لنظرية جرين وناخدي. ويجب الإشارة إلي أن جميع الأشكال والنتائج التي تم الحصول عليها كانت باستخدام برنامج (MATLAB R2013a).

الكلمات المفتاحية: المرونة الحرارية المعممة. المجال المغناطيسي، الإجهاد الأساسي الهيدروستاتيكي، طريقة تحليل الوضع الطبيعي، نظرية (جرين وناخدي).

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