



Roughness in Membership Continuous Function

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Abstract: In this paper, we introduce the new definition of rough membership function using continuous function and we discuss several concepts and properties of rough continuous set value functions as new results on rough continuous function and membership continuous function. Moreover, we extend the definition of rough membership function to topology spaces by substituting an equivalence class by continuous functions and prove some theorems on certain types of set value functions and some more general and fundamental properties of the generalized rough sets. Our result generalized the concept of the set valued function by using rough set theory.

Keywords: Rough set; Lower approximation; Upper approximation; Set valued mapping; membership continuous function.

INTRODUCTION

The theory of rough set has been introduced by Pawlak (Pawlak, 1982). It was introduced as new mathematical method in an incomplete information. Recently, many researchers have used the rough theory in itself and many areas in the real-life applications. However, other research found the connection between rough sets and many areas such as algebraic systems (Biwas & Nanda, 1994; Davvaz, 2004; Pawlak & Skowron, 2007). The set valued functions have been used in many areas such as Economics (Aubin & Frankowska, 2009; Davvaz, 2006, 2008; Vind, 1964).

Here, we rewrite the definition of rough membership function by using continuous function and we discuss several concepts and properties of continuous set value functions as new results on rough the continuous function. We introduce a new definition of rough membership function in topology spaces by continuous functions and we give proofs of relevant theorems and fundamental properties.

The lower and upper approximations are defined as follows:

Definition 1-1. A set valued function $F: X \rightarrow P(X)$ is function from non-empty X to $P(X)$ the set of all non -empty subsets of X such that $F(x) \neq \emptyset$ for all $x \in X$. If $B \subset X$, then we define the upper rough approximation by $\overline{F(B)} = \{x \in X | F(x) \subset B\}$ and the lower rough approximation by $\underline{F(B)} = \{x \in X | F(x) \cap B \neq \emptyset\}$. Therefore $(\overline{F(B)}, \underline{F(B)})$ is called F-rough set of X . The boundary is $B(B) = \overline{F(B)} - \underline{F(B)}$, if $B \neq \emptyset$, then $B(B)$ is rough.

Remark 1-1. We define the domain of F by $DF = \{x \in X : F(x) \neq \emptyset\}$, and the graph of F by $Graph(F) = \{(x, y) : y \in F(x)\}$. So, the image of F is a subset of X defined by $Im(F) = \bigcup_{x \in X} F(x) = \bigcup_{x \in DF} F(x)$.

Note that, if we define the domain of F by $DF = \{x \in X : F(x) \neq \emptyset\}$, the set-valued map F is characterized by its graph; $Graph(F) = \{(x, y) : y \in F(x)\}$, and the domain of F is the projection of $Graph(F)$ on X . The image of F is a subset of X defined by $Im(F) = \bigcup_{x \in X} F(x) =$

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$\bigcup_{x \in D_F} F(x)$. It is the projection of $Graph(F)$ on X .

Remark 1-2 If $F: X \rightarrow P(X)$, then we call upper semi-continuous mapping on X if the set $\overline{F(A)}$ (resp. $\underline{F(A)}$) is closed in X where A is closed in X .

Remark 1-3: If $F: X \rightarrow P(X)$, then we call upper semi-continuous mapping on X if the set $\overline{F(A)}$ (resp. $\underline{F(A)}$) is open in X where A is open in X .

Example 1-1: let $X = \{1, 2, 3, 4, 5, 6\}$ and let $F : X \rightarrow P(X)$ where for every $x \in X$, $F(1) = \{1\}$, $F(2) = \{1, 3\}$, $F(3) = \{3,4\}$, $F(4) = \{4\}$, $F(5) = \{1,6\}$, $F(6) = \{1, 5, 6\}$. Let $A = \{1, 3, 5\}$, then $\overline{F(A)} = \{1, 2\}$, and $\underline{F(A)} = \{1, 2, 3, 5, 6\}$, $B(A) \neq \emptyset$, is rough. $Im_{(F)} = \bigcup_{x \in X} F(x) = \{1,3,4,5,6\}$. Let $B = \{2,4,6\}$ then $\overline{F(B)} = \{4\}$, and $\underline{F(B)} = \{3,4, 5, 6\}$, $B(B) \neq \emptyset$, is rough.

Definition 1-2: Suppose that $F: X \rightarrow P(X)$ is a set valued function. We define the upper continuous if for all $x \in X$ and any open $V \subset P(X)$ contain $F(x)$, then there is an open $O \subset X$ contain x such that $F(O) \subset V$. And the lower continuous if for any $x \in X$ and for any open $V \subset P(X)$ such that $F(x) \cap V \neq \emptyset$ than there is open $O \subset X$ contain x such that $F(O) \cap V \neq \emptyset$. Therefore, we say F is continuous if and only F has this property at each point of X .

Definition 1.4. Let $F: X \rightarrow P(X)$ be a set-valued function and A be an event in the function approximation space $S = (X, P)$. Then the lower probability of A is $P_{\text{alpbability}}(A) = P_{\text{alpbability}}(\overline{F(A)})$, and the upper probability is $P^{\text{alpbability}}(A) = P_{\text{alpbability}}(\underline{F(A)})$.

Note that, respectively. Clearly, $0 \leq P_{\text{alpbability}}(A) \leq 1$ and $0 \leq P^{\text{alpbability}}(A) \leq 1$.

Example 1-2: we consider example 1-1, for $A = \{1, 3, 5\}$ the upper $\overline{F(A)} = \{1, 2\}$, then $P_{\text{alpbability}}(\overline{F(A)}) = 2/6$ and the lower

$\underline{F(A)} = \{1, 2, 3, 5, 6\}$, then $P_{\text{alpbability}}(\underline{F(A)}) = 5/6 = 1$

Proposition 1.1. Let $F: X \rightarrow P(X)$ be a set-valued function and A, B be two events in the stochastic approximation space $S = (X, P)$. Then the following holds:

- (1) $P^{\text{alpbability}}(\emptyset) = \emptyset = P_{\text{alpbability}}(\emptyset)$;
- (2) $P^{\text{alpbability}}(X) = 1 = P_{\text{alpbability}}(X)$;
- (3) $P_{\text{alpbability}}(AB) \leq P_{\text{alpbability}}(A) + P_{\text{alpbability}}(B) - P_{\text{alpbability}}(A \cap B)$;
- (4) $P_{\text{alpbability}}(A \cup B) \leq P_{\text{alpbability}}(A) + P_{\text{alpbability}}(B) - P_{\text{alpbability}}(A \cap B)$;
- (5) $P_{\text{alpbability}}(Ac) = 1 - P_{\text{alpbability}}(A)$;
- (6) $P_{\text{alpbability}}(AB) \leq P_{\text{alpbability}}(A) - P_{\text{alpbability}}(A \cap B)$;
- (7) $P_{\text{alpbability}}(A) \leq P_{\text{alpbability}}(A)$;
- (8) If $A \subseteq B$, then $P_{\text{alpbability}}(A) \leq P_{\text{alpbability}}(B)$ and $P_{\text{alpbability}}(A) \leq P_{\text{alpbability}}(B)$.

Proof. It is unpretentious

Definition 1-5 : Suppose that $F: X \rightarrow P(X)$ be a set-valued function. Let A be an event in the stochastic approximation space $S = (X, P)$. The rough probability of A , denoted by $P^*(A)$, is given by: $P^*(A) = (P_{\text{alpbability}}(A), P_{\text{alpbability}}(A))$.

Proposition 1.2: Let $F: X \rightarrow P^*(X)$ be a set-valued function and A be an event in the stochastic approximation space $S = (X, P)$.

- (1) If F has reflective, then $P_{\text{alpbability}}(A) \leq P(A) \leq P_{\text{alpbability}}(A)$;
- (2) If F has reflective and transitive properties, then $P_{\text{alpbability}}(\overline{F(B)}) = P_{\text{alpbability}}(A)$ and $P_{\text{alpbability}}(\underline{F(A)}) = P_{\text{alpbability}}(A)$;
- (3) If A is an exact subset of X , then $P_{\text{alpbability}}(A) = P_{\text{alpbability}}(A) = P(A)$.

Proof. It is unpretentious.

Rough of membership continuous set valued function.

The rough membership function has been defined by equivalence class (Davvaz, 2004). In

addition, (Lashin et al., 2005) extended the definition of rough membership function to topology spaces. Pawlak and Skowron (Pawlak & Skowron, 1993), introduced the concept of rough membership functions as a tool for reasoning with uncertainty.

We introduce the new definition of rough membership function using the semi-continuous function $F(x)$ as:

$$\mu_A^{F(x)}(x) = \frac{|F(x) \cap A|}{|F(x)|}, F(x) \in P(X), x \in X \dots \dots (*)$$

Definition 2-1: $A \subseteq X$, closure of A is \bar{A} and A° is interior, and A^b is boundary. A is exact if $A^b = \emptyset$, otherwise A is rough. A is exact iff $\bar{A} = A^\circ$.

Example 2-1: consider example 1-1, we have $\bar{F(A)} = \{1, 2\}$, and $\underline{F(A)} = \{1, 2, 3, 5, 6\}$,

$\bar{F(B)} = \{4\}$, and $\underline{F(B)} = \{3, 4, 5, 6\}$,

We can see, $A \cap B = \emptyset$. $\bar{F(A \cap B)} = \emptyset$, and $\underline{F(A \cap B)} = \emptyset$.

Let $C = \{1, 2, 3\}$, then $\bar{F(C)} = \{1, 2\}$, and $\underline{F(C)} = \{1, 2, 3, 4, 5, 6\}$, $A \cap C = \{1, 3\}$.

$\bar{F(A \cap B)} = \{1, 2\}$, and $\underline{F(A \cap B)} = \{1, 2, 3, 5, 6\}$. $P-(A/C) = 2/2 = 1$, $P+(A/C) = 5/6$,

$$\begin{aligned} \mu_{A \cap C}^{F(x)}(1) &= \frac{|F(x) \cap \{1, 3\}|}{|\{1, 3\}|} = \frac{1}{1} = 1 & ; \mu_{A \cap C}^{F(x)}(2) &= \frac{|F(x) \cap \{1, 3\}|}{|\{1, 3\}|} = \frac{2}{2} = 1; \mu_{A \cap C}^{F(x)}(3) &= \frac{|F(x) \cap \{1, 3\}|}{|\{3, 4\}|} = \frac{1}{1} \\ \mu_{A \cap C}^{F(x)}(4) &= \frac{|F(x) \cap \{1, 3\}|}{|\{4\}|} = \frac{0}{1} = 0 \\ \mu_{A \cap C}^{F(x)}(5) &= \frac{|F(x) \cap \{1, 3\}|}{|\{1, 6\}|} = \frac{1}{2} \\ \mu_{A \cap C}^{F(x)}(6) &= \frac{|F(x) \cap \{1, 3\}|}{|\{1, 5, 6\}|} = \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \mu_A^{F(x)}(1) &= \frac{|F(x) \cap \{1, 3, 5\}|}{|\{1\}|} = \frac{1}{1} = 1 & ; \mu_A^{F(x)}(2) &= \frac{|F(x) \cap \{1, 3, 5\}|}{|\{1, 3\}|} = \frac{2}{2} = 1; \mu_A^{F(x)}(3) &= \frac{|F(x) \cap \{1, 3, 5\}|}{|\{3, 4\}|} = \frac{1}{1} \\ \mu_A^{F(x)}(4) &= \frac{|F(x) \cap \{1, 3, 5\}|}{|\{4\}|} = \frac{0}{1} = 0 \end{aligned}$$

$$\mu_A^{F(x)}(5) = \frac{|F(x) \cap \{1, 3, 5\}|}{|\{1, 6\}|} = \frac{1}{2}$$

$$\mu_A^{F(x)}(6) = \frac{|F(x) \cap \{1, 3, 5\}|}{|\{1, 5, 6\}|} = \frac{2}{3}$$

If $B = \{2, 4, 6\}$

$$\mu_B^{F(x)}(1) = \frac{|F(x) \cap \{2, 4, 6\}|}{|\{1\}|} = 0 & ; \mu_B^{F(x)}(2) &= \frac{|F(x) \cap \{2, 4, 6\}|}{|\{1, 3\}|} = 0; \mu_B^{F(x)}(3) &= \frac{|F(x) \cap \{2, 4, 6\}|}{|\{3, 4\}|} = \frac{1}{2}$$

$$\mu_B^{F(x)}(4) = \frac{|F(x) \cap \{2, 4, 6\}|}{|\{4\}|} = \frac{1}{1} = 1$$

$$\mu_B^{F(x)}(5) = \frac{|F(x) \cap \{2, 4, 6\}|}{|\{1, 6\}|} = \frac{1}{2}$$

$$\mu_B^{F(x)}(6) = \frac{|F(x) \cap \{2, 4, 6\}|}{|\{1, 5, 6\}|} = \frac{1}{3}$$

Now, for $s A \cup B = X$

$$\mu_X^{F(x)}(1) = \frac{|F(x) \cap X|}{|\{1\}|} = \frac{1}{1} = 1 & ; \mu_X^{F(x)}(2) &= \frac{|F(x) \cap X|}{|\{1, 3\}|} = \frac{2}{2} = 1; \mu_X^{F(x)}(3) &= \frac{|F(x) \cap X|}{|\{3, 4\}|} = \frac{2}{2} = 1$$

$$\mu_X^{F(x)}(4) = \frac{|F(x) \cap X|}{|\{4\}|} = \frac{1}{1} = 1$$

$$\mu_X^{F(x)}(5) = \frac{|F(x) \cap X|}{|\{1, 6\}|} = 1; \mu_X^{F(x)}(6) = \frac{|F(x) \cap X|}{|\{1, 5, 6\}|} = \frac{3}{3} = 1$$

$$\mu_X^{F(x)}(6) = \frac{|F(x) \cap X|}{|\{1, 5, 6\}|} = \frac{3}{3} = 1$$

$$\mu_X^{F(x)}(6) = \frac{3}{3} = 1$$

Then we have $\mu_X^{F(x)} = \sum \mu_A^{F(x)} + \mu_B^{F(x)} = 6$

We conclude the next proposition;

Proposition 2.1

If β is a family of pairwise disjoint subsets of X then $\mu_{\cup \beta}^{F(x)}(x) = \sum_{A \in \beta} \mu_A^{F(x)}(A)$ for any $x \in X$.

Proof

We have $\mu_{\cup \beta}^{F(x)}(x) = \frac{|F(x) \cap \cup \beta|}{|F(x)|} =$

$$\frac{|\cup \{F(x) \cap A : A \in \beta\}|}{|F(x)|} = \sum_{A \in \beta} \mu_A^{F(x)}(A)$$

Remark 2-1: we can define the fuzzy set by using the equation (*) $\check{A} = \{(x, \mu_A^{F(x)}(x))\}$.

Form example 1-1, for Let $A = \{1, 3, 5\}$, we can define $\check{A} = \{(1, 1), (2, 1), (3, 1), (4, 0), (5, 1/2), (6, 2/3)\}$.

However, the rough membership (*) is very different from rough set theory or Lashin's rough membership function (Sedghi et al., 2017).

Proposition 2-2. Let $A \subseteq X$. The rough membership function $\mu_A^{F(x)}(x)$ has the following properties:

$$\mu_A^{F(x)}(x) = 1 \text{ iff } x \in \overline{F(A)};$$

$$\mu_A^{F(x)}(x) = 0 \text{ iff } x \in \underline{F(A)}^c.$$

Proof

If $x \in \overline{F(A)}$; then $x \in X \mid F(x) \subseteq A$, then $\mu_A^{F(x)}(x) = 1$.

If $\mu_A^{F(x)}(x) = 1$, then $x \in \overline{F(A)}$;

We have $x \in \underline{F(A)}^c \Leftrightarrow F(x) \cap A = \emptyset \Leftrightarrow \mu_A^{F(x)}(x) = 0$.

Example 2-2: From example 2-1 let $X = \{1, 2, 3, 4, 5, 6\}$, Let $A = \{1, 3, 5\}$, then $\overline{F(A)} = \{1, 2\}$, and $\underline{F(A)} = \{1, 2, 3, 5, 6\}$,

$$\mu_A^{F(x)}(1) = \frac{|\{1\} \cap \{1,3,5\}|}{|\{1\}|} = \frac{1}{1} = 1 \quad ; \mu_A^{F(x)}(2) = \frac{|\{1,3\} \cap \{1,3,5\}|}{|\{1,3\}|} = \frac{2}{2} = 1;$$

$$\mu_A^{F(x)}(3) = \frac{|\{3,4\} \cap \{1,3,5\}|}{|\{3,4\}|} = \frac{0}{1} = 0$$

$$; \mu_A^{F(x)}(4) = \frac{|\{4\} \cap \{1,3,5\}|}{|\{4\}|} = \frac{0}{1} = 0$$

$$; \mu_A^{F(x)}(5) = \frac{|\{1,6\} \cap \{1,3,5\}|}{|\{1,6\}|} = \frac{1}{2};$$

$$\mu_A^{F(x)}(6) = \frac{|\{1,5,6\} \cap \{1,3,5\}|}{|\{1,5,6\}|} = \frac{2}{3}.$$

Let $B = \{2,4,6\}$ then $\overline{F(B)} = \{4\}$, and $\underline{F(B)} = \{3,4,5,6\}$.

$$\mu_B^{F(x)}(1) = \frac{|\{1\} \cap \{2,4,6\}|}{|\{1\}|} = 0 \quad ; \mu_B^{F(x)}(2) = \frac{|\{1,3\} \cap \{2,4,6\}|}{|\{1,3\}|} = 0;$$

$$\mu_B^{F(x)}(3) = \frac{|\{3,4\} \cap \{2,4,6\}|}{|\{3,4\}|} = \frac{1}{2}$$

$$; \mu_B^{F(x)}(4) = \frac{|\{4\} \cap \{2,4,6\}|}{|\{4\}|} = \frac{1}{1} = 1$$

$$; \mu_B^{F(x)}(5) = \frac{|\{1,6\} \cap \{2,4,6\}|}{|\{1,6\}|} = \frac{1}{2};$$

$$\mu_B^{F(x)}(6) = \frac{|\{1,5,6\} \cap \{2,4,6\}|}{|\{1,5,6\}|} = \frac{1}{3}.$$

We can extend the concepts of rough set approximations to any subfamily of $P(X)$.

Definition 2-2: Let $\eta \subseteq P(X)$, we define the η -upper approximation $\overline{F(\eta)} = \{A \in P(B) \mid F(A) \subseteq \eta\}$ and η -lower approximation $\underline{F(\eta)} = \{A \in P(X) \mid F(A) \cap \eta \neq \emptyset\}$.

Proposition 2-3:

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$$F(F(x)) = \{F(x)\} \text{ for all } x \in X.$$

Proof:

We have $\overline{F(F(x))} = F(x) = \overline{F(F(x))}$;

if, then $\underline{F(A)} = \underline{F(F(x))} = F(x) = \overline{F(F(x))} = \overline{F(A)} = A$. Therefore, $F(F(x)) = \{F(x)\}$.

Approximation of continuous set valued mapping and membership function rough topology.

We introduce the new definition of rough membership function using the semi-continuous function extending the definition of rough membership function to topology spaces by (Lashin et al., 2005).

$$\mu_A^\tau(x) = \frac{|F(x) \cap A|}{|F(x)|}, \quad F(x) \in P(X), x \in X \dots \dots \dots (*)$$

Example 3-1: let $X = \{1, 2, 3, 4, 5, 6\}$ and let $F : X \rightarrow P(X)$ where for every $x \in X$, $F(1) = \{1,2,3\} = F(2)$, $F(3) = \{3,4\} = F(4)$, $F(5) = \{4,5\}$, $F(6) = \{6\}$. Let $A = \{1, 2, 3, 4\}$,

Then $S = \{\{1,2,3\}, \{3,4\}, \{4,5\}, \{6\}\}$, then $\beta = \{\{1,2,3\}, \{3,4\}, \{4,5\}, \{6\}, \{3\}, \{4\}\}$,
We get $\tau = \{X, \emptyset, \{1,2,3\}, \{3,4\}, \{4,5\}, \{6\}, \{3\}, \{4\}, \{1,2,3,4\}, \{1,2,3,4,5\}, \{1,2,6\}, \{3,4,5\}, \{3,4,6\}, \{6\}, \{4,6\}, \{3,4,5,6\}\}$.

$$\mu_A^\tau(1) = \frac{|\{1,2,3\} \cap \{1,2,3,4\}|}{|\{1,2,3\}|} = \frac{1}{1} = 1 \quad ; \mu_A^\tau(2) = 1;$$

$$\mu_A^\tau(3) = \frac{|\{3,4\} \cap \{1,2,3,4\}|}{|\{3,4\}|} = \frac{2}{2} = 1 \quad ; \mu_A^\tau(4) = 1$$

$$; \mu_A^\tau(5) = \frac{|\{4,5\} \cap \{1,2,3,4\}|}{|\{4,5\}|} = \frac{1}{2}; \quad \mu_A^\tau(6) = \frac{|\{6\} \cap \{1,2,3,4\}|}{|\{6\}|} = \frac{0}{1} = 0.$$

Let $B = \{5, 6\}$, we have $\mu_B^\tau(1) = \frac{|\{1,2,3\} \cap \{5,6\}|}{|\{1,2,3\}|} = 0$; $\mu_B^\tau(2) = 0$; $\mu_B^\tau(3) = 0$; $\mu_B^\tau(4) = 0$

$$; \mu_B^\tau(5) = \frac{|\{4,5\} \cap \{5,6\}|}{|\{4,5\}|} = \frac{1}{2}; \quad \mu_B^\tau(6) = \frac{|\{6\} \cap \{5,6\}|}{|\{6\}|} = \frac{1}{1} = 1.$$

For X .

$$\mu_X^\tau(1) = \frac{|\{1,2,3\} \cap X|}{|\{1,2,3\}|} = 1 = \mu_X^\tau(2);$$

$$\mu_X^\tau(3) = \frac{|(\{3,4\} \cap X)|}{|\{3,4\}|} = \frac{2}{2} = 1 = \mu_X^\tau(4)$$

$$; \mu_X^\tau(5) = \frac{|(\{4,5\} \cap X)|}{|\{4,5\}|} = 1; \mu_X^\tau(6) = \frac{|(\{6\} \cap X)|}{|\{6\}|} = 1 .$$

Proposition 3.1

Suppose that β is a family of pairwise disjoint subsets of X then $\mu_{\cup\beta}^\tau(x) = \sum_{A \in \beta} \mu_A^\tau(A)$ for any $x \in X$.

Proof

The same way of proof in theory Proposition 2.1.

Note that, we can get the interior and closure of A from the family F of all τ -closed sets:

$$F = \{A, \emptyset, \{4,5,6\}, \{1,2,5,6\}, \{1,2,3,6\}, \{1,2,3,4,5\}, \{1,2,4,6\}, \{1,2,3,5,6\}, \{5,6\}, \{6\}, \{4,5\}, \{1,2,6\}, \{1,2,6\}, \{1,2,5\}, \{1,2,3\}, \{1,2,4,5\}, \{1,2,3,5\}, \{1,2\}\}$$

$$A^\circ = \{1,2,3\} \cup \{3,4\} \cup \{3\} \cup \{4\} = \{1,2,3,4\}, \bar{A} = X \cap \{1,2,3,4,5\} = \{1,2,3,4,5\}.$$

Note that we can get it from rough membership function.

$\underline{F}(\bar{A}) = \{1, 2, 3, 4\}$, and $\underline{F}(A) = \{1, 2, 3, 4, 5\}$, it is clear A is rough from definition 2-1. Also, $B(A) \neq \emptyset$, then rough.

CONCLUSION

The rough sets theory is considered as a generalization of the classical sets theory. The main idea of rough set was built by equivalence relations. Occasionally, an equivalence is difficult to be obtained in rearward problems due to vagueness and incompleteness of human knowledge. We generalized a rough set theory in continuous functions and substituted an equivalence class by continuous functions. Moreover, we introduced a new definition of rough membership function using continuous function and discussed several concepts and properties of rough continuous set value functions as new results on rough the continuous function and membership continuous function. In addition, we extended the

definition of rough membership function to topology spaces by substituting an equivalence class with continuous functions. Our result connects rough sets, topology spaces, fuzzy sets, and semi continuous function. We believe our result has many applications in some areas.

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الغموض في الدالة العضوية المتصلة التقريبية

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المستخلص: تقدم الورقة تعريفاً جديداً لدالة العضوية التقريبية باستخدام الدالة المستمرة، وتناقش العديد من المفاهيم، والخصائص لدوال القيم المستمرة التقريبية كنتائج جديدة على الدالة التقريبية، والدوال العضوية المستمرة. علاوة على ذلك، تقدم الورقة توسعا لتعريف دالة العضوية التقريبية إلى فضاءات الطوبولوجيا عن طريق استبدال فئة التكافؤ بدوال مستمرة، وثبتت بعض النظريات حول أنواع معينة من الدوال القيمة المحددة، وبعض الخصائص الأساسية، والعمومية للمجموعات الخشنة المعممة. النتائج المتحصل عليها عممت مفهوم دالة المجموعة القيمية باستخدام نظرية المجموعات الخشنة. تعتبر تعميم الدالة العضوية التقريبية باستخدام دوال المتصلة.

الكلمات المفتاحية: المجموعات الخشنة، التقريب من أعلى، التقريب من أسفل، خرائط مجموعة قيمية، الدالة العضوية المتصلة.